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Math 6 Unit B




Student Learning Guide

we explore



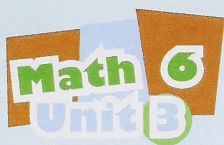
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Math 6 Learn EveryWare – Unit 3 Student Learning Guide
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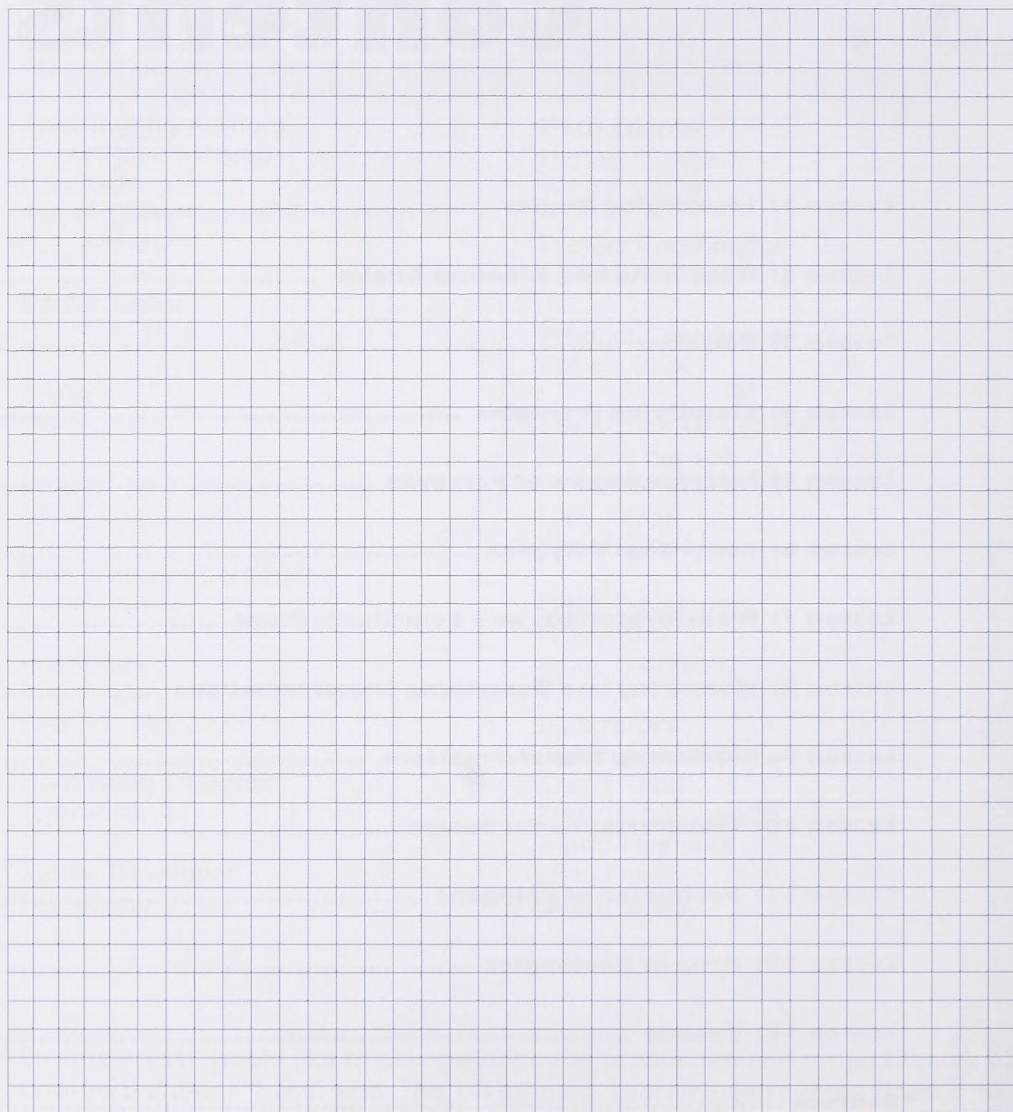
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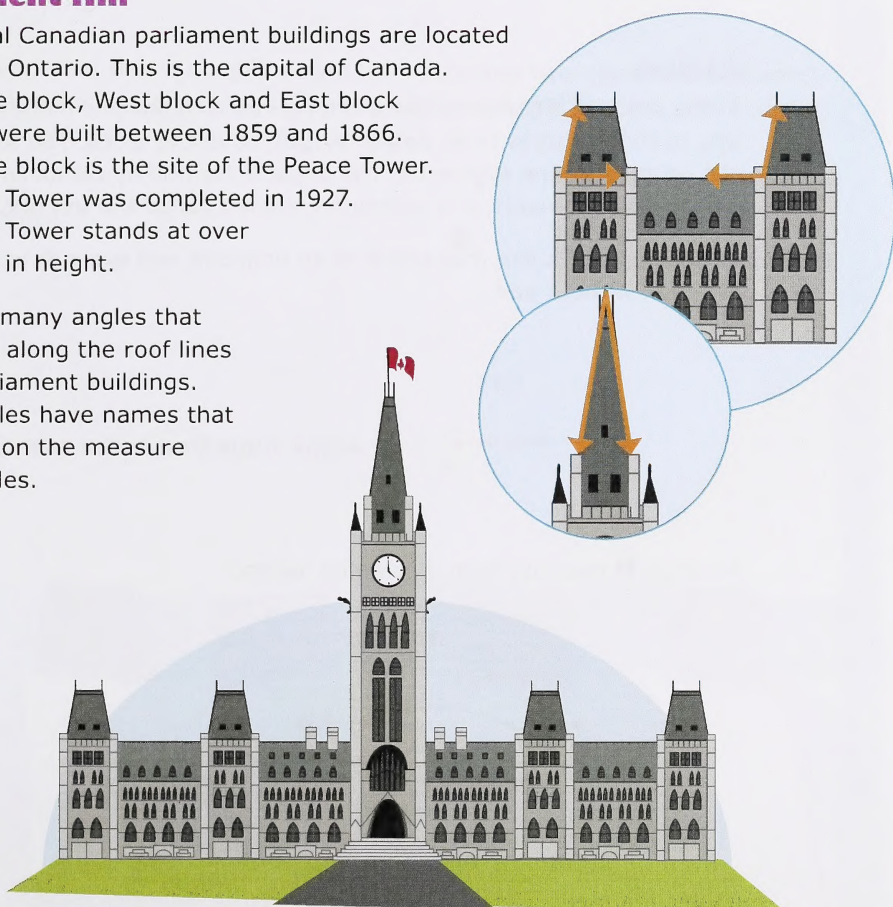
Lesson 1

Identifying Angles

Parliament Hill

The federal Canadian parliament buildings are located in Ottawa, Ontario. This is the capital of Canada. The Centre block, West block and East block buildings were built between 1859 and 1866. The Centre block is the site of the Peace Tower. The Peace Tower was completed in 1927. The Peace Tower stands at over 92 metres in height.

There are many angles that were used along the roof lines of the Parliament buildings. These angles have names that are based on the measure of the angles.



Objectives for this Lesson

In this lesson you will explore the following:

- Estimate the measure of an angle
- Sketch angles
- Provide examples of angles found in the environment
- Classify a given set of angles

Angles

In the study of Geometry, there are figures with specific names. One of these figures is an **angle**. Angles have two sides. The sides can be **rays** or line segments. The angle is formed by combining two rays or line segments at a common endpoint called the **vertex**.

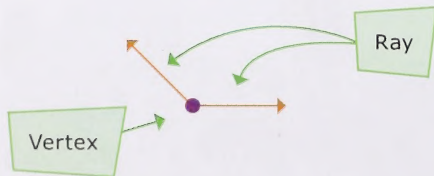
A ray is part of a line that starts at an endpoint and extends in one direction without end.

Ray



An angle is two rays with a common vertex:

Angle



Angles are measured using a unit of measure called degrees. One degree is $\frac{1}{360}$ th of a circle.

There are special angles that you should be able to determine the measure of at a glance.

90° Angle



You may recall that this is a **right angle** from studying two-dimensional shapes such as rectangles and squares. You should also recall that the box in the corner is used to show that an angle is a right angle.

180° Angle



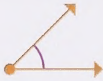
This angle is also called a **straight angle**.

Reflection

Why do you think the 180° angle is called a "straight angle"?

- Go online to watch the Notepad Tutor Lesson: 45° , 90° , and 180° Angles.

45° Angle



135° Angle



You can use these angles to estimate the measure of other angles.

Example 1

Estimate the measure of the angle given:



Use an angle that you know to compare:



Here is a 45° angle on top of the angle. You can see that the angle we need to estimate is a little less than half the size of the 45° angle.

An estimate of the angle is: 20°

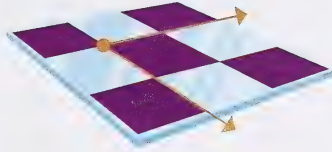
Identifying Angles in the World

You can find angles everywhere around you. Look around your room now. Do you see any angles?

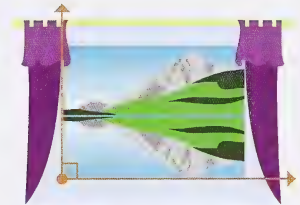
You should be able to find a few examples like these:



Chair



Tile Floor



Window



Let's Explore

Exploration 1: Angles in Your Room

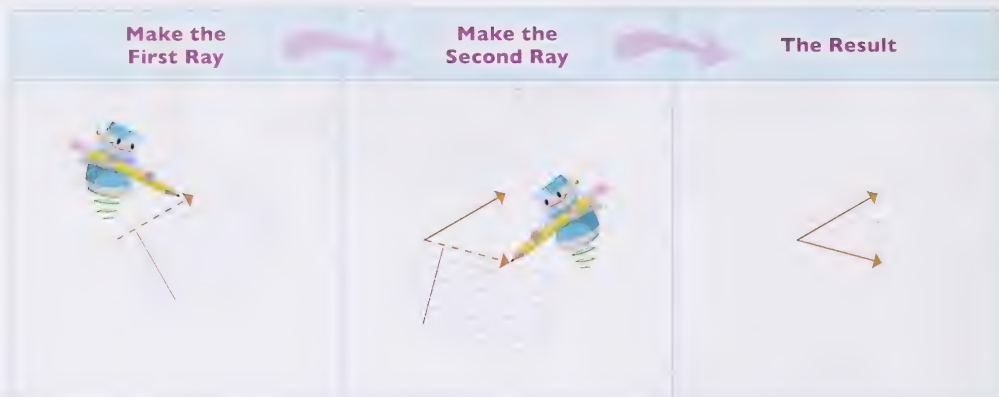
Materials: Unit 3, Lesson 1 Exploration page in your Workbook, Pencil

1. Look around your room and identify three objects that form angles that are approximately 90° , 45° , and 135° . List these objects and their estimated angle measure in the table provided.
2. Look around your room and identify angles that are less than 90° and angles that are greater than 90° . List these objects and their estimated angle measures in the table provided.

Being able to identify these angles in the environment will help you remember angle measures that are common.

Sketching Angles

You can sketch an angle using a **straight edge**. A straight edge is any straight object that you have around you. For example, you could use the side of a note card since the card is heavy. It will not bend when you run your pencil down the side to make a line.



Example 2

Sketch a 45° angle.

Use a straight edge to sketch the angle:



Classifying Angles

Angles can be classified with the following names:

right angle

acute angle

obtuse angle

straight angle

You should recall that right angles measure 90° and straight angles measure 180° .

Right Angle



Straight Angle



Each of the following angles measures less than 90° .
These angles are called acute.



Each of the following angles measures more than 90° and less than 180° .
These angles are called obtuse.



Example 3

Classify each angle as right, acute, obtuse, or straight.

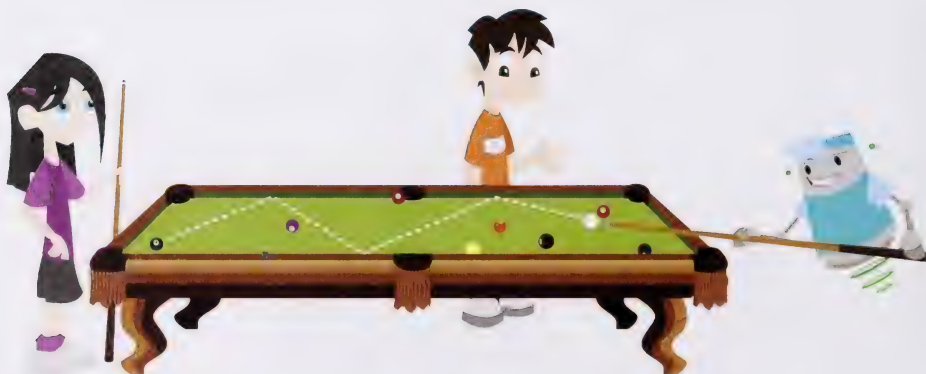


- This angle is larger than 90° and less than 180° so it is **obtuse**.
- This angle is 180° so it is a **straight** angle.
- This angle is 90° so it is a **right** angle.
- This angle is less than 90° so it is **acute**.



Let's Practice

- Turn in your Workbook to Unit 3, Lesson 1 and complete 1 to 15.



Lesson 2

Measuring and Creating Angles

Museums in Canada

There are many beautiful museums in Canada. Visitors go to appreciate the historical items and artwork in the museums. You can see that the museums have beautiful designs to the buildings. Architects are hired to design buildings. They use mathematics to create beautiful designs. These designs draw people to the buildings and help them appreciate the collections and art works inside.

Canada's Royal
Ontario Museum



National Gallery
in Ottawa, Ontario



Reflection

Do you see any angles in these structures?
Are the angles all the same or do they vary?

Objectives for this Lesson

In this lesson you will explore the following concepts:

- Measure given angles in various orientations
- Draw and label a specified angle in various orientations

Measuring Angles

Angles are measured using an instrument called a **protractor**.



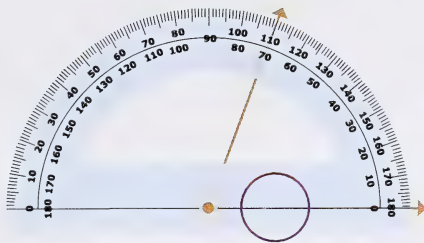
Every angle has a degree measure. There is a process to find the measure of an angle.

Finding an angle measure:

1. Line up the circle on the bottom of the protractor with the vertex of the angle.



- Line up the bottom line with one ray.



- Read the degree measure on the outside scale if the angle opens to the left. Read the degree measure on the inside scale if the angle opens to the right.



This angle is 70° .

Use the following exploration to understand protractors and angle measures.



Let's Explore

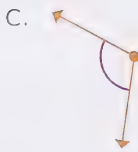
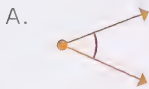
Exploration 1: Protractors and Angles

Materials: Unit 3, Lesson 2, Exploration 1 page in your Workbook, Protractor, Pencil

- Create your own angle. Hint: Use the straight edge of your protractor to make the rays as straight as possible.

2. Measure your angle using a protractor. Record the measure.
3. Reflect: What do you find to be challenging about measuring an angle?
4. Measure the following angles and record your results in the table.

A	
B	
C	
D	



5. Reflect: What can you do to get the most accurate angle measures?



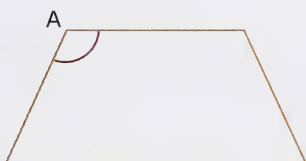
Identifying an Angle

You can use a symbol to say "angle". It looks like an angle.

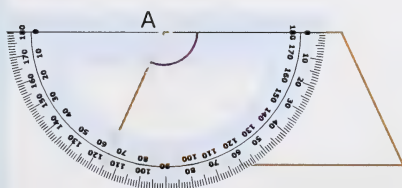
Figure	Name in Words	Name in Symbols
	Angle A	$\angle A$

Example 1

Find the measure of the indicated angle.



Line up your protractor with the vertex of the indicated angle and one side of the angle.



Read using the outside scale since the side that is lined up must start with 0° .

$$\angle A = 115^\circ$$

Drawing an Angle

You can use a protractor to draw an angle of a given measure. The process is only slightly different than measuring angles.

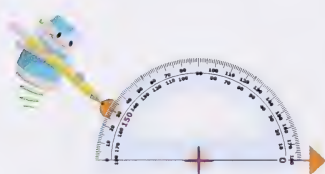
Example 2

Draw an angle that is 150° .

1. Draw one side of the angle.



2. Place the centre point of the protractor on the vertex and line the side up with the 0° mark. Find 150° on the protractor and make a dot there.

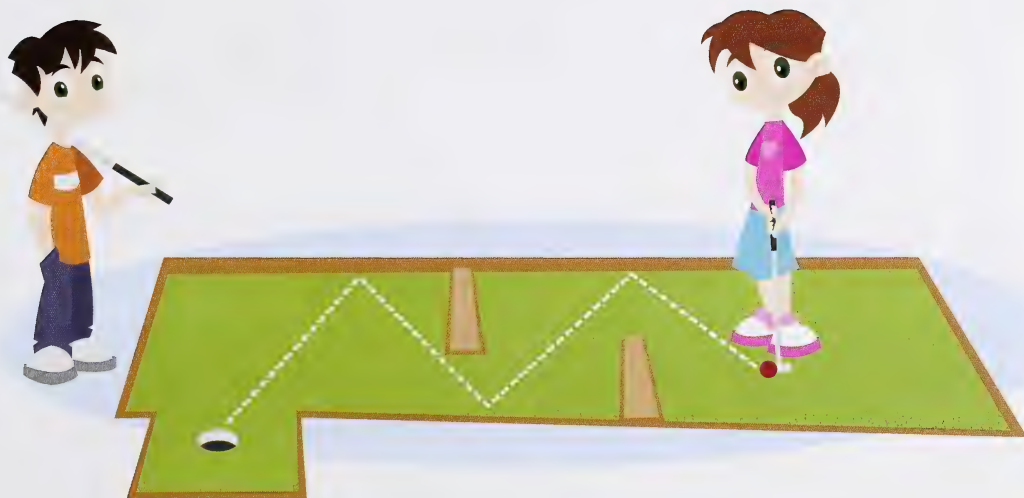


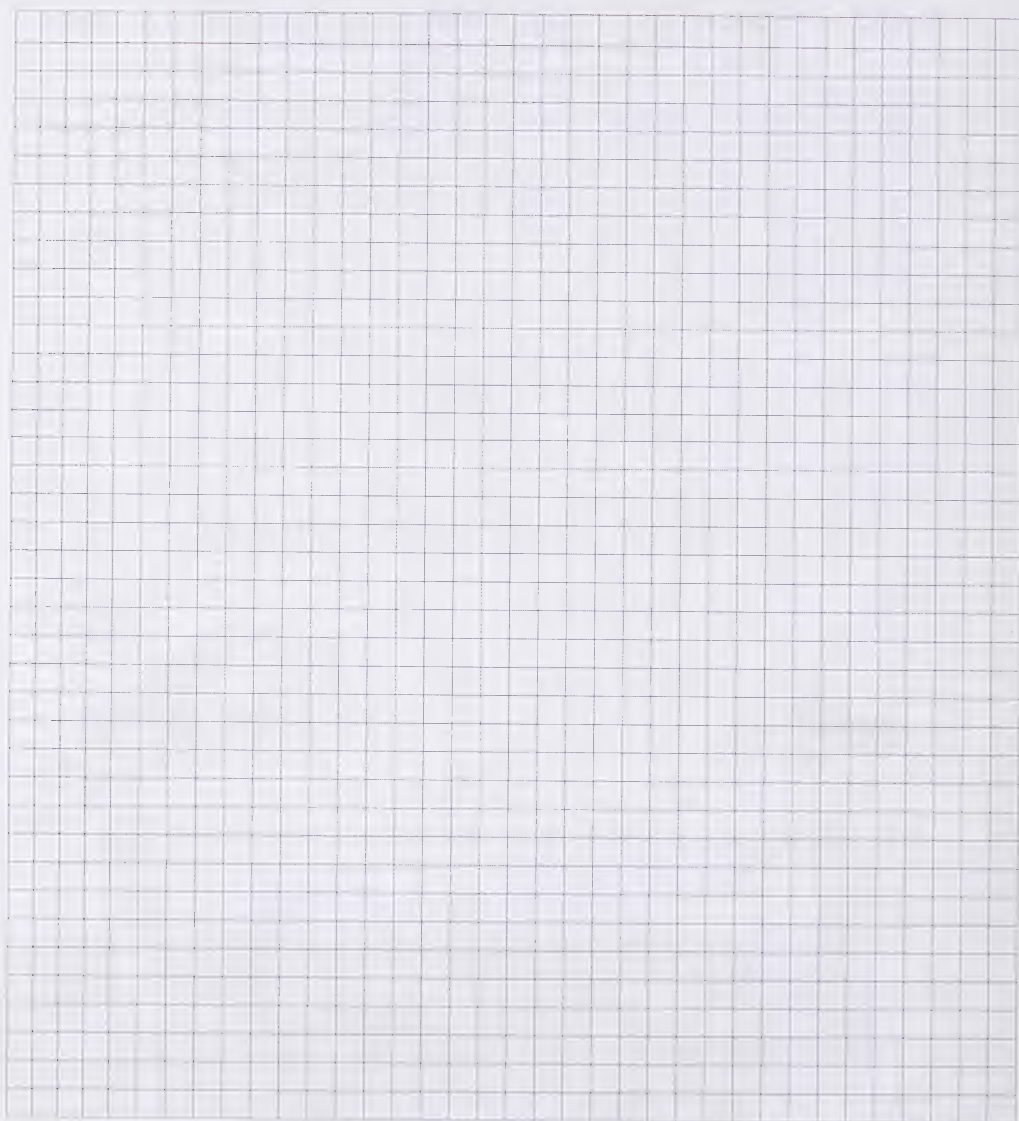
3. Remove the protractor and use the straight side of the protractor to draw the side from the vertex to the dot at 150° .



**Let's Practice**

- Go online to complete the Concept Capsule: Drawing Angles.
- Turn in your Workbook to Unit 3, Lesson 2 and complete 1 to 17.





Lesson 3

Polygons

Honeycombs

Bees make a special container to hold their honey. It is called a honeycomb by honey producers. You can buy honey with the honeycomb in the honey. The honeycomb is made of wax that beekeepers usually return to the nest. They do this because the bees actually have to drink the honey to produce the wax that makes the honeycomb. By putting the comb back in the hive after taking out the honey, they decrease the amount of time it takes for the honeycomb to be filled again.



The honeycomb is made up of compartments in the shape of hexagons. These hexagons are the same length on each of the six sides.

Reflection

Can you think of other geometric shapes that have the same side length on each side?

Objectives for this Lesson

In this lesson you will explore the following concepts:

- The difference between polygons and non-polygons
- Congruence in regular polygons
- The difference between regular or irregular polygons
- Regular and irregular polygons in the environment

- Go online to complete the [Concept Capsule: Classifying Quadrilaterals](#).

Classifying Polygons

You have studied many types of shapes. You can use properties of shapes to write a definition for a group of shapes with the same characteristics.



Let's Explore

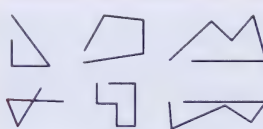
Exploration 1: Classifying Polygons

Materials: Unit 3, Lesson 3, Exploration 1 page in your Workbook, Pencil, Recording Device (optional)

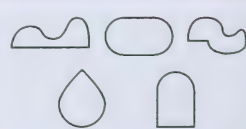
1. Observe the following sets of shapes.



A



B



C

2. What do these shapes have in common?

Sets	Common Traits
A and B	
A and C	
B and C	

3. What is different about these shapes?

Sets	Differences
A and B	
A and C	
B and C	

For 4 – 7: Either record an oral description or write one in your Workbook. If recording, follow the recording instructions in your Workbook.

- Describe the shapes in Set A.
- Describe the shapes in Set B.
- Describe the shapes in Set C.
- The shapes in Set A are called polygons. The shapes in Sets B and C are not polygons. Write or orally record your own definition for **polygon**.

Polygons are closed figures in a plane that have three or more line segments. They do not have any curves.

Congruent Parts of Shapes

The following exploration will help you recall the definition of “congruent”. It will also help you explore congruence.



Let's Explore

Exploration 2: Congruence

Materials: Unit 3, Lesson 3, Exploration 2 page in your Workbook, Scissors, Pencil, Page of shapes from the back of this unit in your Workbook

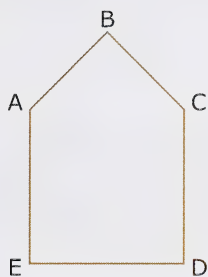
1. Cut out the page of shapes from the back of this unit in your Workbook.
2. Overlay the shapes listed in the table. Compare their size and shape and record your observations in the table.
3. Complete: Two figures are congruent if they are the same _____ and _____.
4. Reflect: Why does placing one shape over another help you determine if two shapes are congruent?

Figures are **congruent** when they have the same size and shape. The symbol \cong means “is congruent to”. You can use it to describe sides and angles in a figure that are congruent.

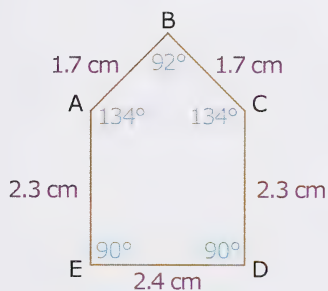
- Go online to watch the [Notepad Tutor Lesson: Corresponding Parts of Congruent Shapes](#).

Example 1

Measure each side and angle in the shape given. Name the congruent parts.



1. Measure and label the parts of the shape.



2. Name the parts that are congruent.

$$\angle A \cong \angle C, \angle E \cong \angle D, AB \cong BC, AE \cong CD$$



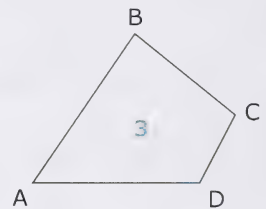


Let's Explore

Exploration 3: Measuring Congruence

Materials: Unit 3, Lesson 3, Exploration 3 page in your Workbook, Centimetre Ruler, Protractor, Pencil

1. Measure each side of the given shapes in centimetres and each angle in degrees. Record the measures in the table.



Shape	Side AB	Side BC	Side CD	Side DA	Angle A	Angle B	Angle C	Angle D
1								
2								
3								

2. Name the parts of Shape 1 that are congruent.
3. Name the parts of Shape 2 that are congruent.
4. Name the parts of Shape 3 that are congruent.
5. Are any of the three figures congruent? Why or why not?

Regular Polygons

Polygons can be either regular or irregular. The following Exploration will help you determine the difference.

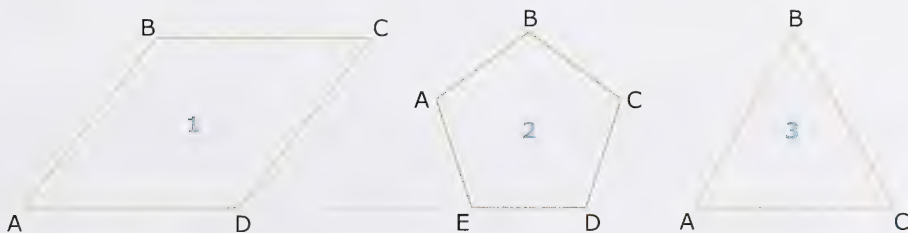


Let's Explore

Exploration 4: Regular or Irregular

Materials: Unit 3, Lesson 3, Exploration 4 page in your Workbook, Centimetre Ruler, Protractor, Pencil

1. Measure each side of the following shapes. Measure each angle in the shapes. Label the figures with each angle measure and side length. Hint: You can extend the lines of the sides with your pencil and the straight edge of your protractor. This will help you to read the angle measure on your protractor.



A **regular polygon** is a polygon with all sides congruent and all angles congruent. An **irregular polygon** has sides and angles of various measures.

2. Which of the shapes are regular polygons?
3. Which of the shapes are irregular polygons?

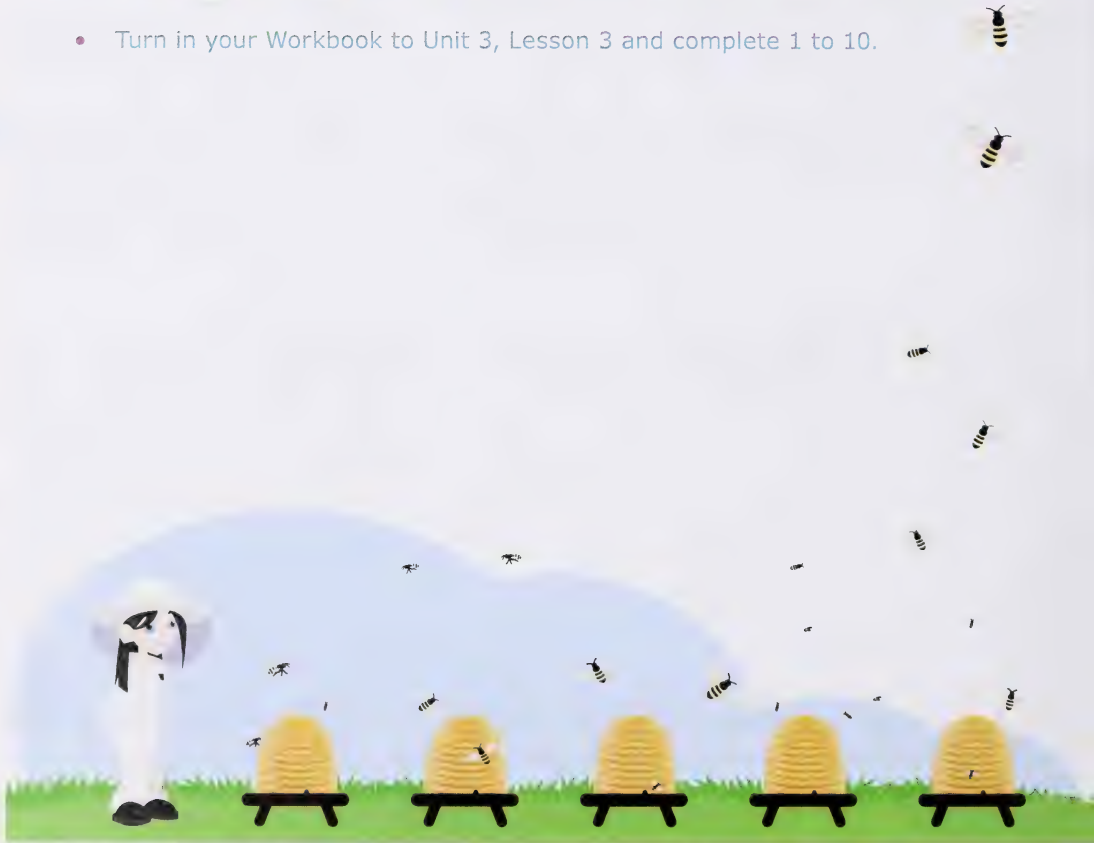
Lesson 3: Polygons

4. Draw two shapes that are regular polygons.
5. How can you tell that your shapes are regular polygons?
6. Reflect: Name one regular polygon in your everyday life.



Let's Practice

- Turn in your Workbook to Unit 3, Lesson 3 and complete 1 to 10.



Lesson 4

Classifying Triangles

Art Sculptures

Sculptures are used as symbols for a message to those that view and admire the work. This sculpture is called Consophia and was created by Ian Lazarus to represent a sharing of knowledge and interaction between the United States and Canada. It is located in Odette Sculpture Park in Windsor, Ontario near the border between the United States and Canada.



The sculpture is made of metal and has three main parts. There is a large triangle, a smaller triangle, and a large horizontal pipe that connects the triangles. It has Ojibway script writing on one side that translates to "inspired interaction". The script on the other side translates to "sharing knowledge".

The triangles have two sides that are the same length.

Reflection

Read the lesson and determine the types of triangles that are used in this sculpture.

Objectives for this Lesson

In this lesson you will explore the following concepts:

- Classify triangles by their sides
- Classify triangles by their angles
- Draw a specified triangle

Classifying Triangles by Sides

A triangle is made up of three line segments. These line segments are joined together at each **endpoint**. The endpoints of the line segments make up the triangle's vertices.



Triangles can be classified by the length of their sides. Use the following Exploration to identify the three types of triangles based on their side measures.



Let's Explore

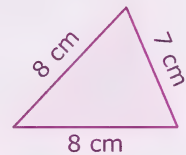
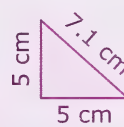
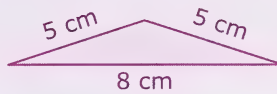
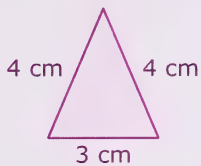
Exploration 1: Classifying Triangles by Sides

Materials: Unit 3, Lesson 4, Exploration 1 page in your Workbook, Centimetre Ruler, Pencil, Recording Device (optional)

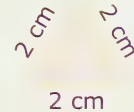
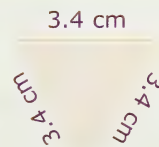
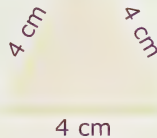
Lesson 4: Classifying Triangles

1. Observe the following sets of triangles.

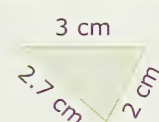
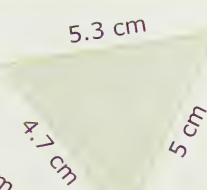
Isosceles Triangles



Equilateral Triangles



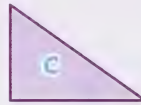
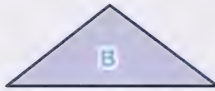
Scalene Triangles



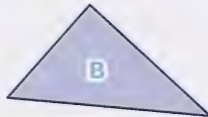
Lesson 4: Classifying Triangles

For 2: Either record your answers or write them in your Workbook.
If recording, follow the recording instructions in your Workbook.

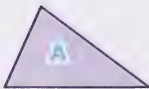
2. What do the triangles in each set have in common?
 - a. **Isosceles triangles**
 - b. **Equilateral triangles**
 - c. **Scalene triangles**
3. Use a centimetre ruler to identify the **isosceles** triangles in this set:



4. Use a centimetre ruler to identify the **equilateral** triangles in this set:



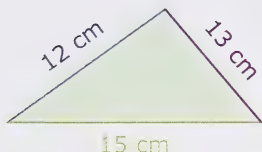
5. Use a centimetre ruler to identify the **scalene** triangles in this set:



Example 1

Classify each triangle by its side lengths.

a.



b.



c.



- No sides are congruent. This triangle is scalene.
- Two sides are congruent. This triangle is isosceles.
- All three sides are congruent. This triangle is equilateral.

- Go online to complete the Concept Capsule: Properties of Triangles.

Classifying Triangles by Angles

You can also classify triangles by the measure of their angles. There are three classifications: **Acute Triangles**, **Obtuse Triangles**, and **Right Triangles**.

An acute triangle has three acute angles. You should recall that an acute angle is less than 90° . Here are a few acute triangles:



Lesson 4: Classifying Triangles

The marks on the sides are used to help you know when sides are the same length. The first triangle has all acute angles and all three sides are the same.



You can classify this triangle by its angles, which are all **acute**, and by its sides, which make it **equilateral**.

That means every triangle can be classified in two ways: by the length of its sides and by its angles.

Triangles Classified by Sides		
Scalene	Isosceles	Equilateral
None of the sides are congruent.	Two sides are congruent.	Three sides are congruent.

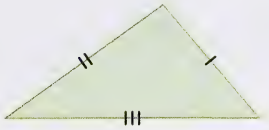
Triangles Classified by Angles		
Acute	Obtuse	Right
All angles are acute. ($< 90^\circ$)	One angle is obtuse. ($> 90^\circ$)	One angle is right. ($= 90^\circ$)

Lesson 4: Classifying Triangles

Example 2

Classify each triangle in two ways.

a.



b.



c.



- This triangle has three different side lengths. It is **scalene**.
All of the angles of this triangle are less than 90° . It is **acute**.
- This triangle has two sides that are congruent. It is **isosceles**.
All of the angles of this triangle are less than 90° . It is **acute**.
- This triangle has two sides congruent so it is **isosceles**.
One angle of this triangle is greater than 90° . It is **obtuse**.



Let's Explore

Exploration 2: Obtuse and Right Triangles

Materials: Unit 3, Lesson 4, Exploration 2 page in your Workbook, Straws, String, Centimetre Ruler, Scissors, Protractor, Pencil

For 1 – 5: Use the materials listed to help you answer the questions.

- Can you create a triangle with two obtuse angles? Explain.
- Can you create a triangle with two right angles? Explain.
- Can you create a triangle that is isosceles and right? Explain.
- Can you create a triangle that is equilateral and obtuse? Explain.
- Can you create a triangle that is isosceles and obtuse? Explain.

Example 3

Draw a triangle that is right and scalene.

1. Make your right angle using a straight edge. Make the sides two different lengths.



2. Join the endpoints of the sides of the right angle.



3. Mark the right angle and each side.



Let's Practice

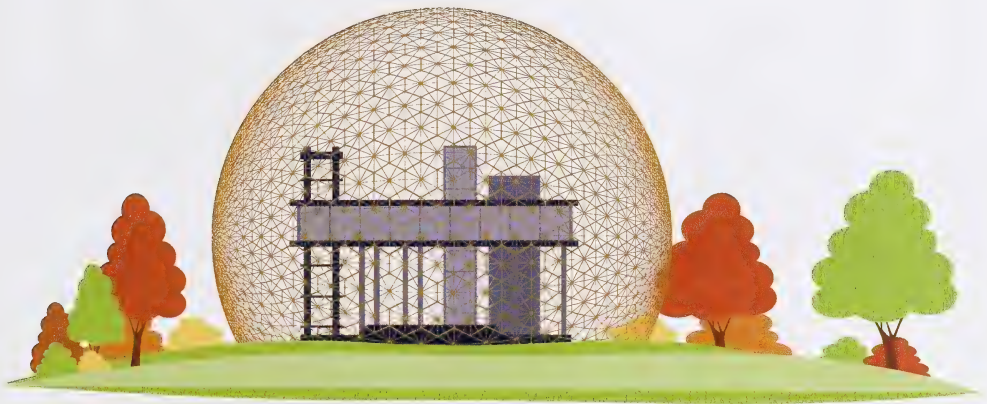
- Turn to your Workbook to Unit 3, Lesson 4 and complete 1 to 11.

Lesson 5

Interior Angles of Polygons

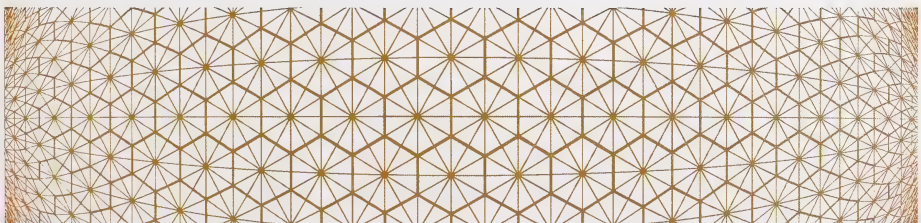
Triangles and the Biosphère

Richard Buckminster Fuller (1895 – 1983) was an engineer, inventor, and architect. One of his greatest works of architecture was the Biosphère that he designed as an exhibit for the 1967 World Fair in Montreal.



The Biosphère was purchased in 1990 by Environment Canada. It is now a museum showcasing environmental exhibits and interactive activities.

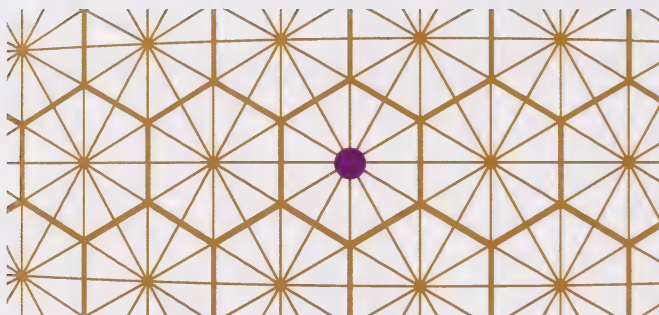
The sphere is made up of many triangles. Fuller believed the triangle was the perfect form to use to create his sphere. He used triangles turned in many ways to create the spherical shape.



Lesson 5: Interior Angles of Polygons

Reflection

In this image, all of the angles that meet at the purple point are the same measure. Is this enough information to find the measure of each angle?



Objects for this Lesson

In this lesson you will explore the following concepts:

- The sum of the angles of a triangle
- The sum of the angles of a quadrilateral

Angles of a Triangle

You have classified triangles by their sides and angles. This gave you an understanding of some of the properties of triangles. Another property of triangles you need to explore is the relationship among the angles of a triangle.



Let's Explore

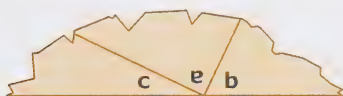
Exploration 1: Angles in a Triangle

Materials: Unit 3, Lesson 5, Exploration 1 page in your Workbook, Scissors, Straight Edge, Protractor, Pencil, Paper

1. Cut out the triangles from the page in your Workbook.
2. Tear each corner from the triangle labelled 1.



3. Position the torn angles so that the vertex of each angle is together and the sides are lined up:



4. Measure the angle formed by the three parts together. What type of angle is formed by the three angles placed together like this?
5. Repeat these instructions with triangles 2 and 3.
6. Draw two more triangles and repeat the instructions.
7. Complete the statement:

The angles of a triangle add up to _____ degrees.

You can find the measure of the missing angle in a triangle. All you need are the measures of at least two angles. Variables are used to mark missing angle measures.

Example 1

Find the measure of the missing angle.



You should be able to solve this one in your head. Here are the details for one method of finding the missing angle.

The angle marked with the box is a right angle so it measures 90° . The sum of the two angles:

$$90^\circ + 35^\circ = 125^\circ$$

If you add the remaining angle, the sum should be 180° . Subtract the sum from 180° to find the missing angle:

$$180^\circ - 125^\circ = 55^\circ$$

The missing angle, a , is 55° .

Example 2

Write an equation and find the missing angle measure.



To write an equation, simply add all three angles to get a sum of 180° :

$$x + 40 + 25 = 180$$

Now solve:

$$x + 40 + 25 = 180$$

1. Add the two angles on the left:

$$x + 65 = 180$$

2. Subtract 65 from both sides:

$$x + 65 - 65 = 180 - 65$$

3 Write the answer in degrees:

$$x = 115$$

$$x = 115^\circ$$

Angles of a Quadrilateral

The angles of a quadrilateral have a special sum similar to the angles of a triangle. Use the following exploration to find the sum of the angles of a quadrilateral.

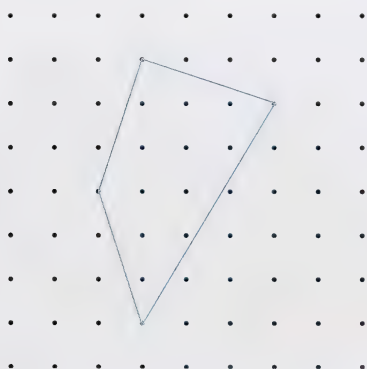


Let's Explore

Exploration 2: Angles of a Quadrilateral

Materials: Unit 3, Lesson 5, Exploration 2 page in your Workbook, Dot Paper from the back of this Unit in your Workbook, Pencil, Protractor

1. Create a quadrilateral on your dot paper that looks like this:



- Measure each angle of the quadrilateral and record them in the table. Find the sum of the angles and record them in the final column of the table.
- Create five more quadrilaterals of your own. Record the measures of the angles and find the sum of the angles.
- What do you notice about the sum of the angles for each quadrilateral?
- Complete this statement:

The sum of the angles in a quadrilateral is _____ degrees.

You can use this conjecture to find the missing angle in a quadrilateral.

Example 3

Find the missing angle.



The three angles that are known: $118 + 108 + 90 = 316$

The final angle will be found by subtracting the sum from 360° : $360 - 316 = 44$

The missing angle, b , is 44° .

Example 4

Find the missing angles.



The two angles that are known: $125(2) = 250^\circ$

The missing angles: $360 - 250 = 110$

Lesson 5: Interior Angles of Polygons

There are two missing angles that are the same measure. You know they are the same measure because they are marked with the same variable. The sum of these angles is 110° .

$$2a = 110$$

$$\frac{2a}{2} = \frac{110}{2}$$

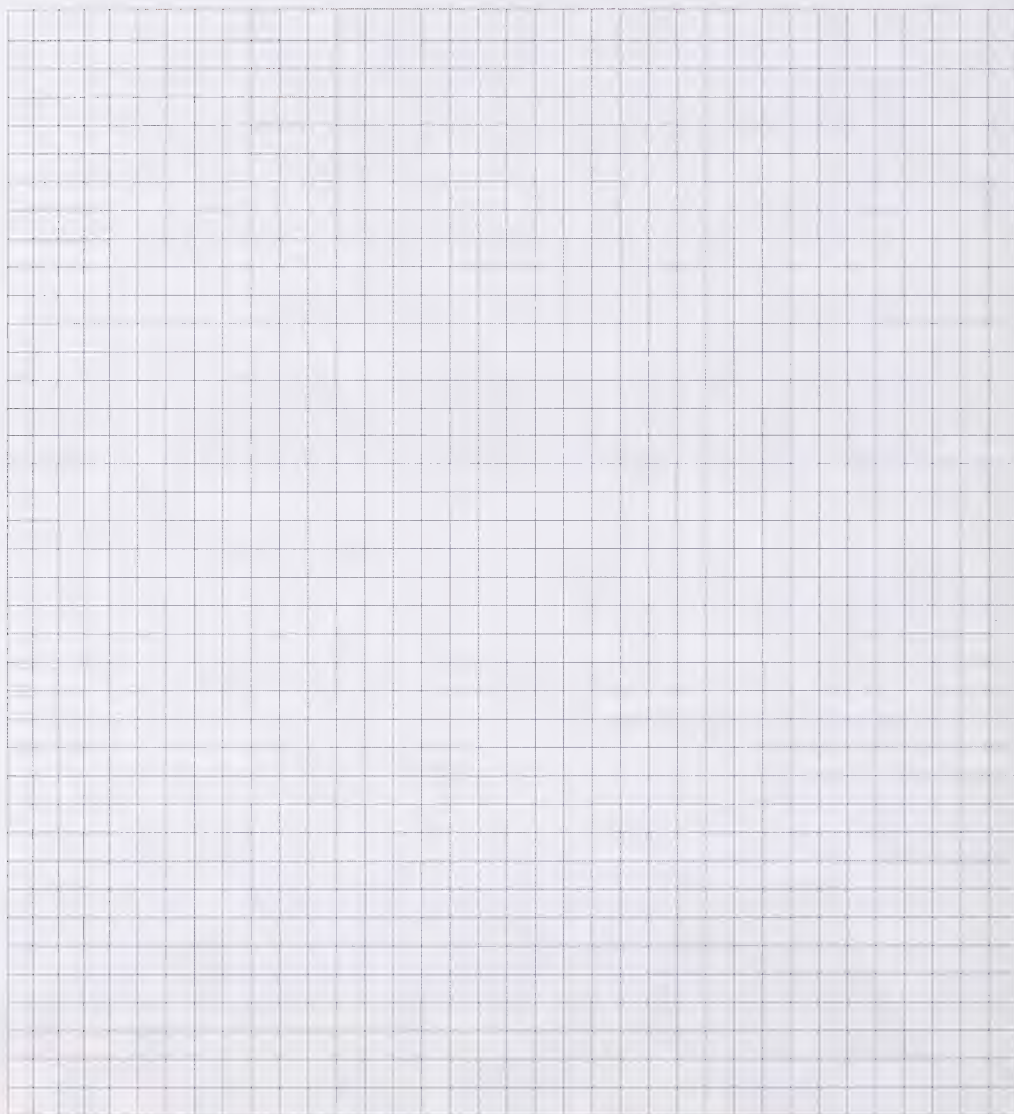
$$a = 55^\circ$$

You should now be able to find the missing angles in a triangle or a quadrilateral.

Let's Practice

- Go online to complete the Concept Capsule: Interior Angles of Triangles and Quadrilaterals.
- Turn in your Workbook to Unit 3, Lesson 5 and complete 1 to 16.





Lesson 6

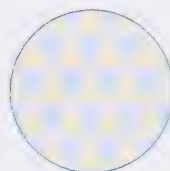
Congruent Polygons

Geometric Art

M.C. Escher (1898 – 1972) was an artist who was inspired by simple geometric formations called **tessellations**. He used simple tessellations to make ones that were much more complex. He would start with a simple tessellation, like this one of equilateral triangles.

He would then create more complex images from that. Here is an example of a tessellation created in the style of M.C. Escher.

The orange fish in this tessellation are all congruent. The blue fish are also congruent.



Reflection

Where in your home do you have congruent figures that make a tessellation?

Objectives for this Lesson

In this lesson you will explore the following concepts:

- Superimposing polygons to show congruence
- Measuring parts of polygons to show congruence
- Congruent polygons in different orientations

Congruent Figures

You should recall that figures are **congruent** if they have the same size and shape. If two polygons have the same size and shape then their corresponding parts are also congruent.



Let's Explore

Exploration 1: Congruent Parts

Materials: Unit 3, Lesson 6, Exploration 1 page in your Workbook, Scissors, Pencil, Page of Shapes (Exploration 1) from the back of this unit in your Workbook, Paper

1. Cut out all of the shapes from your page.
2. Overlap the shapes to find those that are congruent.
3. Name the congruent shapes.
4. How can you tell that your shapes are congruent?
5. Name the corresponding angles for each pair of congruent shapes.
6. Create two shapes of your own that are congruent. Describe how you made your shapes.

- Go online to complete the Concept Capsule: [Congruence in a Regular Polygon](#).

You can also use measuring devices to determine if two figures are congruent.



Let's Explore

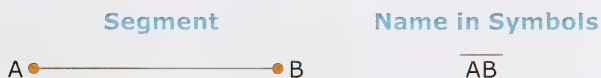
Exploration 2: Measuring Quadrilaterals

Materials: Unit 3, Lesson 6, Exploration 2 page in your Workbook, Pencil, Protractor, Centimetre Ruler, Page of Shapes (Exploration 2) from the back of this unit in your Workbook

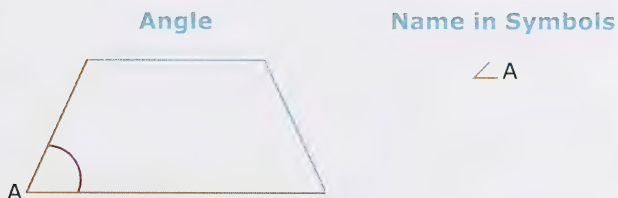
1. Measure the angles and side lengths of each figure on your Exploration page. Record the results in the given table.
2. Which figures are congruent? How can you tell?

Naming Sides and Angles

The sides of a polygon are all segments. You can name segments with a special symbol. A bar over two capital letters is used to name a specific segment.



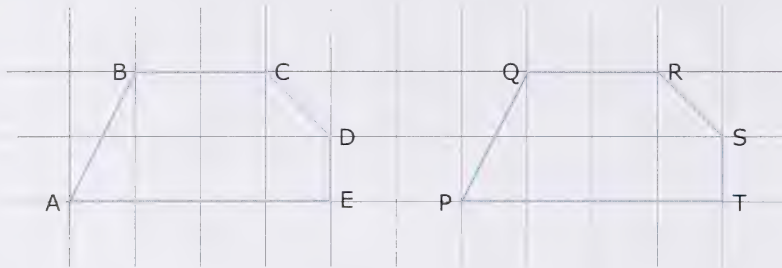
The angles of a polygon are named using the angle symbol.



When you name the congruent parts of congruent shapes you should use these notations.

Example 1

Name the corresponding parts of the congruent figures.



Name all congruent angles:

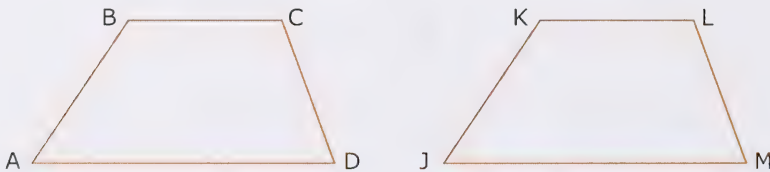
$$\angle A \cong \angle P, \angle B \cong \angle Q, \angle C \cong \angle R, \angle D \cong \angle S, \angle E \cong \angle T$$

Name all congruent segments:

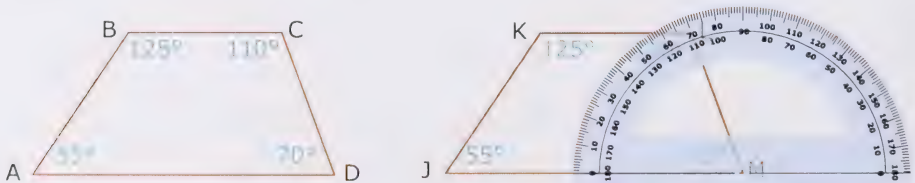
$$\overline{AB} \cong \overline{PQ}, \overline{BC} \cong \overline{QR}, \overline{CD} \cong \overline{RS}, \overline{DE} \cong \overline{ST}, \overline{AE} \cong \overline{PT}$$

Example 2

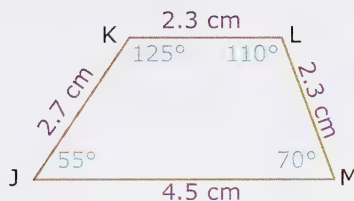
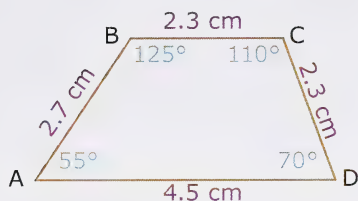
Decide if the figures are congruent by measuring the corresponding parts. Use a centimetre ruler and a protractor.



Measure and label corresponding angles:



Measure and label corresponding sides:



All corresponding sides and angles are congruent. This means:

$$ABCD \cong JKLM$$

Different Orientations

The smiley faces are congruent.

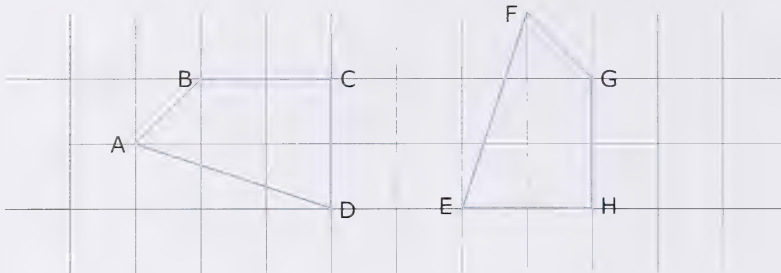


They are the same size and shape, but you should have noticed they are not going in the same direction. This is called **orientation**. They each have a different orientation.

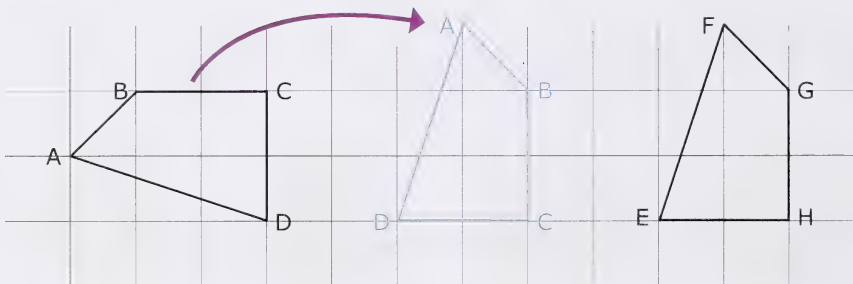
You should be able to identify corresponding parts for congruent figures even when they have a different orientation.

Example 3

Name the congruent parts of the congruent figures.



Notice that if you turn $ABCD$ it will have the same orientation as $FGHE$:



Name all congruent angles:

$$\angle A \cong \angle F, \angle B \cong \angle G, \angle C \cong \angle H, \angle D \cong \angle E$$

Name all congruent sides:

$$\overline{AB} \cong \overline{FG}, \overline{BC} \cong \overline{GH}, \overline{CD} \cong \overline{HE}, \overline{DA} \cong \overline{EF}$$



Let's Practice

- Turn in your Workbook to Unit 3, Lesson 6 and complete 1 to 12.

Lesson 7

Transformations on a Coordinate Plane

Symmetry in Architecture

The Taj Mahal is located in Agra, India. It was built by the Mughal Emperor of India. His name was Shan Jahan. He wanted to build the Crown Palace as a memorial to his beloved wife.



Reflection

Do you see the many lines of symmetry in the picture of the Taj Mahal?




Objectives for this Lesson

In this lesson you will explore the following concepts:

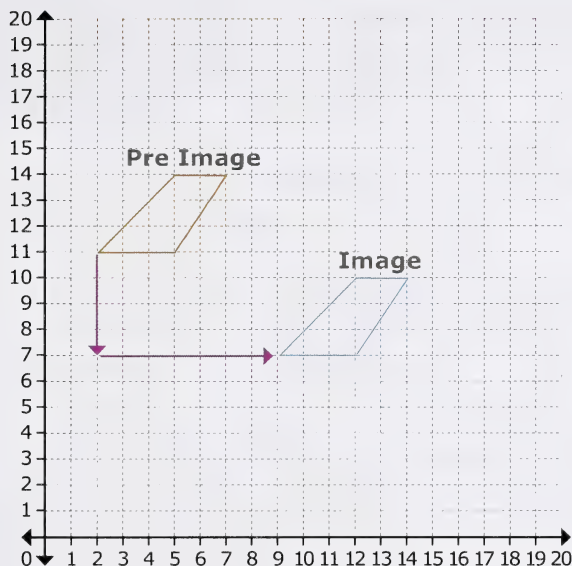
- Draw shapes or designs in the first quadrant of a coordinate plane
- Identify the points used to produce the designs in a coordinate plane
- Perform a transformation on a given 2-D shape, and identify the coordinates of the vertices of the image
- Go online to complete the Concept Capsule: Naming Transformations.

Basic Transformations

You should recall the basic types of **transformations**.

Translation	Reflection	Rotation
 <p>Moves all points in a figure the same distance and direction.</p>	 <p>Moves each point of a figure the same distance over a line.</p>	 <p>Turns each point of a figure around a given point.</p>

You can use a coordinate plane to perform transformations. The original figure is called the **pre-image**. After the transformation is performed the result is the **image**.



Let's Explore

Exploration 1: Digital Transformations

Materials: Unit 3, Lesson 7, Exploration 1 page in your Workbook, Internet, Pencil

1. Go to the website: <http://www.learnalberta.ca/content/memng/index.html>
2. On the left, select **Grades 4 to 6**.
3. Scroll down and select **Reflection (Flip)**.
4. Use the Demonstration Applet until you are familiar with reflections.
This activity will help you explore reflections. Answer the following questions:
 - a. What do you notice about the distance of each shape from the line?

- b. When you move the pre-image closer to the line, what happens to the image?
 - c. What did you notice about the size and shape of the pre-image and the image?
 - d. What did you notice about the orientation of the two shapes?
5. On the left, scroll down and select **Translation (Slide)**.
6. Use the Demonstration Applet until you are familiar with translations. This activity will help you explore translations. Answer the following questions:
 - a. What happens when you change the length of the line between the shapes?
 - b. Why does the image change when you change the pre-image?
7. On the left, scroll down and select **Rotation (Turn)**.
8. Use the Demonstration Applet until you are familiar with rotations. This activity will help you explore rotations. Answer the following questions:
 - a. What is the difference between the pre-image and its image after a rotation?
 - b. Which shape(s) appear to be a reflection under a $\frac{1}{2}$ turn?
 - c. What type of rotation puts the image on top of the pre-image?





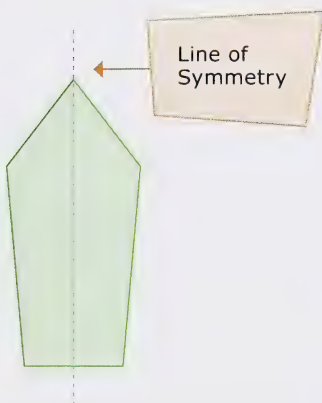
Let's Explore

Exploration 2: Transformations on the Coordinate Plane

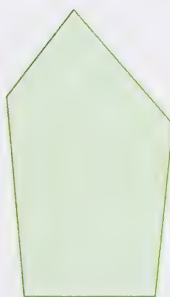
Materials: Unit 3, Lesson 7, Exploration 2 page in your Workbook, Pencil, Straight Edge, Paper, Scissors

1. Create your own shape on the coordinate plane. Make sure that your shape is **asymmetrical**. This means that you cannot draw a line of symmetry on the shape.

Symmetrical



Asymmetrical

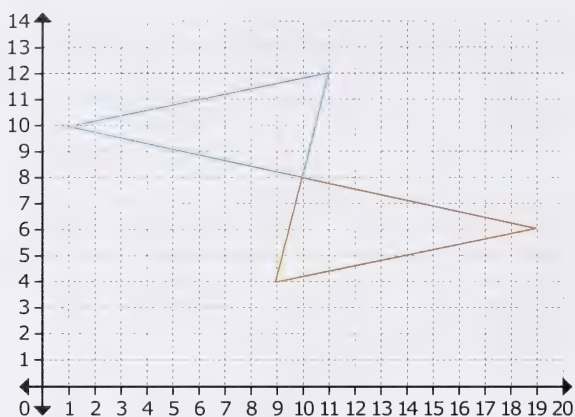


2. Trace your shape on a separate piece of paper. Cut out the tracing. You can use this duplicate of your shape to perform transformations on the coordinate plane.
3. Perform a **translation** of your shape. Identify the coordinates of each vertex on the pre-image and the image.
4. Perform a **rotation** of your shape. Identify the coordinates of each vertex of the pre-image and the image.
5. Perform a **reflection** of your shape. Identify the coordinates of each vertex of the pre-image and the image.

You should be able to determine where the points of a pre-image have moved when a transformation is performed.

Example 1

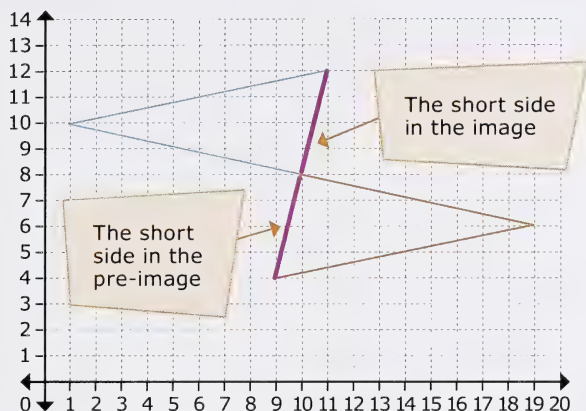
The image is shown in blue. Name the corresponding coordinates of the vertices for the pre-image and the image. What type of transformation is shown?



List the vertices of the pre-image first:

Pre-image	Image
(9, 4)	
(10, 8)	
(19, 6)	

Now look at the position of the images. Choose one side of the pre-image and try to visualize where that side is in the image.



You may have to repeat that visualization with the other sides. When you can see in your head the way the figure has moved, you are ready to list corresponding vertices.

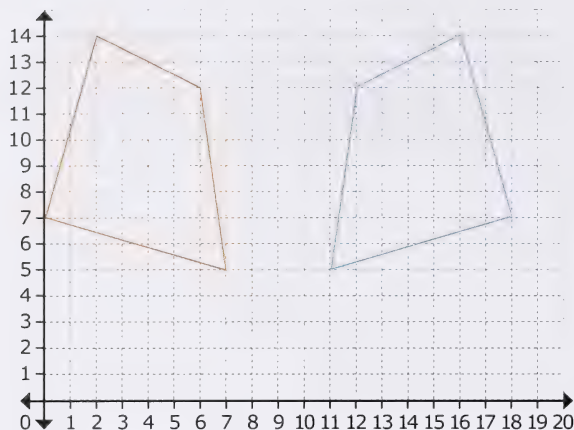
Pre-image	Image
(9, 4)	(11, 12)
(10, 8)	(10, 8)
(19, 6)	(1, 10)

This is a **rotation** of the pre-image.

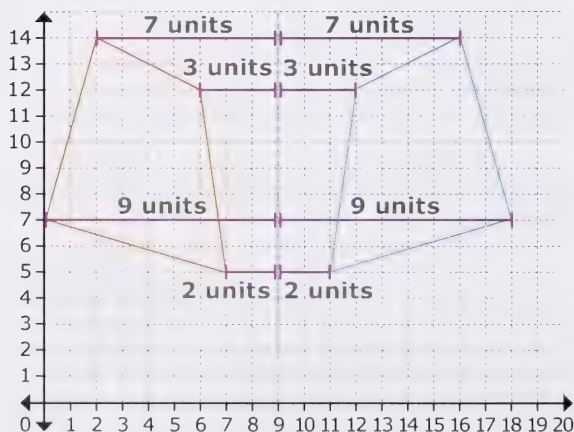
When you identify a rotation, you should be able to tell the point about which the figure was rotated on the coordinate plane. In this case it is a vertex of both image and pre-image. The rotation is about the point (10, 8). This is called the **point of rotation**.

Example 2

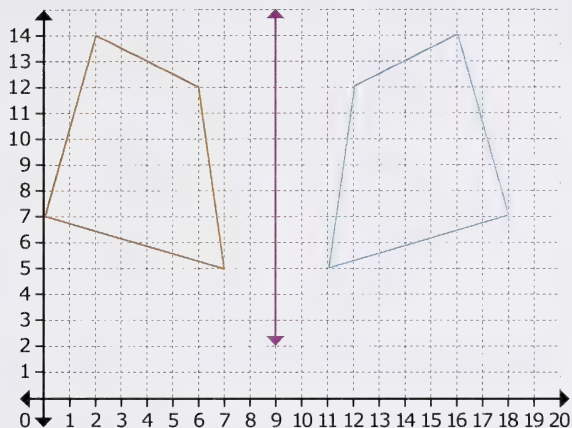
Draw the line of reflection for this image and pre-image.



The line of reflection is halfway between the corresponding points.

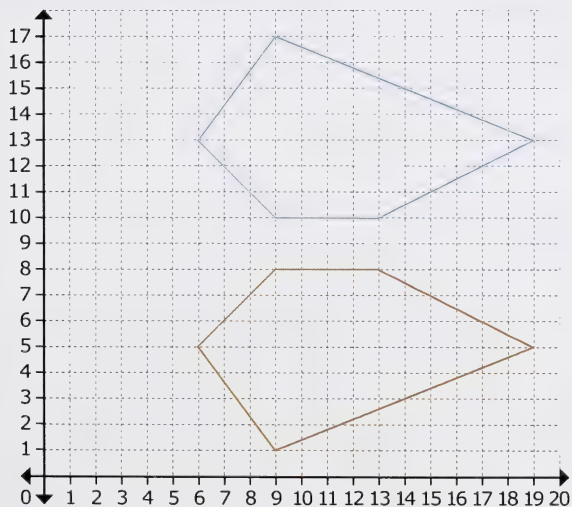


Draw the line of reflection when you have found the halfway point of each pair of corresponding points. This is a reflection through a vertical line.



Example 3

The image is shown in blue. Name the coordinates of the vertices of the image.
What type of transformation is shown?



Identify the vertices of the pre-image:

Pre-image	Image
(6, 5)	
(9, 8)	
(13, 8)	
(19, 5)	
(9, 1)	

Now identify the corresponding vertices of the image:

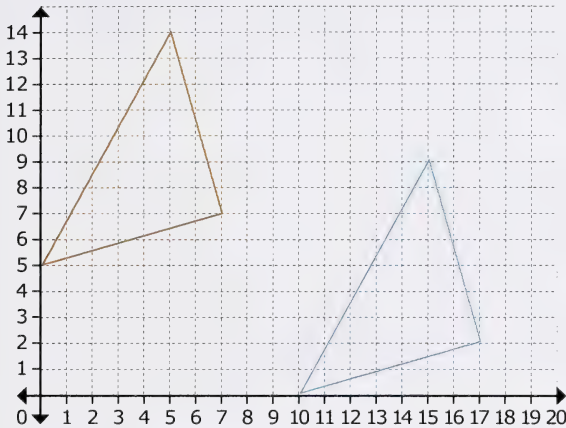
Pre-image	Image
(6, 5)	(6, 13)
(9, 8)	(9, 10)
(13, 8)	(13, 10)
(19, 5)	(19, 13)
(9, 1)	(9, 17)

This is a **reflection through a horizontal line**.

- Go online to watch the Notepad Tutor Lesson: Reflections in First Quadrant of a Cartesian Plane.

Example 4

The image is shown in blue. Name the coordinates of the vertices of the image. What type of transformation is shown?



List the vertices of the pre-image:

Pre-image	Image
(10, 0)	
(15, 9)	
(17, 2)	

Now notice that the orientation of the image is the same. List the corresponding vertices of the pre-image:

Pre-image	Image
(10, 0)	(0, 5)
(15, 9)	(5, 14)
(17, 2)	(7, 7)

This is a **translation**.



Let's Practice

- Go online to watch the Notepad Tutor Lesson: Translations in First Quadrant of a Cartesian Plane.
- Turn in your Workbook to Unit 3, Lesson 7 and complete 1 to 10.



Lesson 8

Measuring and Describing Transformations

Wheels

Cars and trucks can have some pretty interesting wheels. There are a variety of types that can be chosen. Many wheels have a type of symmetry called **rotational symmetry**. A figure with rotational symmetry has a point of rotation and the figure is repeated at a given angle about that point of rotation.



Reflection

Describe the rotational symmetry in one of these wheels.



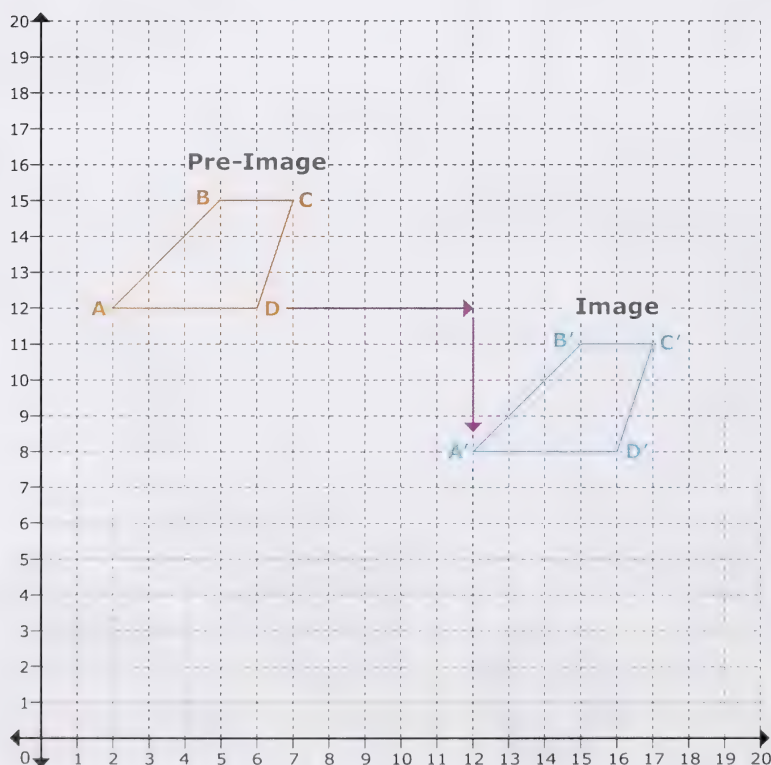
Objectives for this Lesson

In this lesson you will explore the following concepts:

- Demonstrate that a shape and its transformation image are congruent
- Describe the transformations performed on a 2-D shape to produce a given image
- Describe the positional change of the vertices of a given shape to the corresponding vertices of its image as a result of a transformation

Measuring Transformations

Pre-images are often named with letters on the vertices (A, B, etc.). The corresponding vertices on the images are sometimes named with the same letter and a **prime** symbol on the letter name (A' , B' , etc.). This helps you see how the points have moved during the transformation.



This shows that A has moved to A' (A prime), B has moved to B' , C to C' , and D to D' .



Let's Explore

Exploration 1: Measuring Transformations on a Coordinate Plane

Materials: Unit 3, Lesson 8, Exploration 1 page in your Workbook, Coordinate Planes from the back of this unit in your Workbook, Pencil Crayons, Protractor, Centimetre Ruler

1.
 - a. Use one colour to create a quadrilateral with the following vertices:
A (1, 4), B (3, 8), C (8, 8), D (6, 4). Quadrilateral ABCD is the pre-image.
 - b. Use another colour to create a quadrilateral with the following vertices:
A' (6, 11), B' (8, 15), C' (13, 15), D' (11, 11). This is the image of Quadrilateral ABCD.
 - c. Measure the segments of both the pre-image and the image. Measure the angles of both the pre-image and the image. Record the measures in centimetres or degrees.
 - d. What type of transformation has been performed on Quadrilateral ABCD?
 - e. What do you notice about the measures of the segments and the angles?

2.
 - a. Use another coordinate plane to graph the following pre-image and image. Use a different colour for each figure.

A (2, 13), B (9, 19), C (7, 13)

A' (14, 11), B' (7, 5), C' (9, 11)
 - b. Measure and record the angle measures and side lengths of Triangle ABC and its image.
 - c. What type of transformation has been performed on Triangle ABC?
 - d. What do you notice about the measures of the segments and the angles?

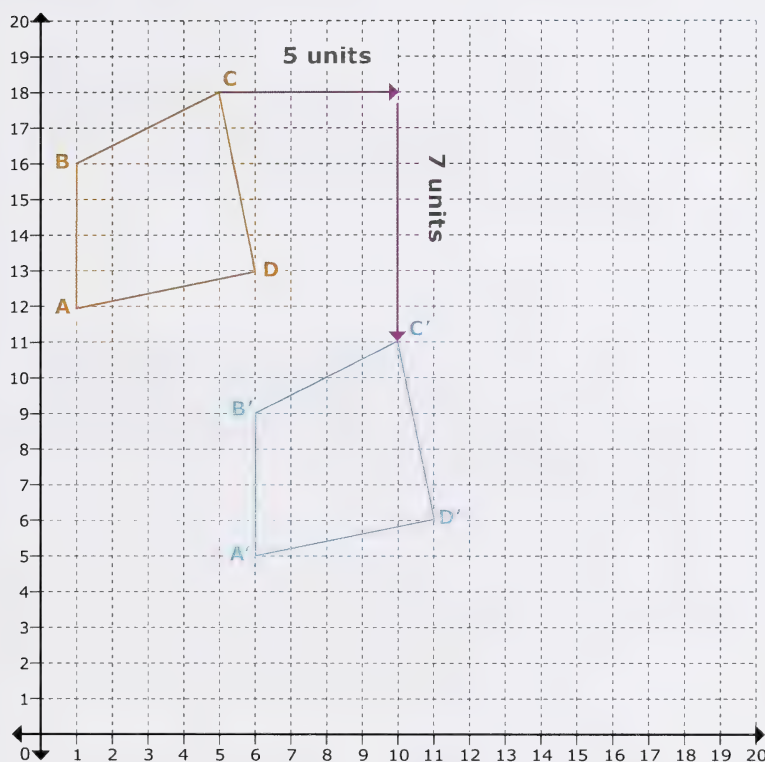
3. a. Use another coordinate plane to graph the following pre-image and image.
Use a different colour for each figure.

 $A (2, 10), B (0, 16), C (9, 17), D (7, 11)$
 $A' (18, 10), B' (20, 16), C' (11, 17), D' (13, 11)$
 - b. Measure and record the angle measures and side lengths of Quadrilateral ABCD and its image.
 - c. What type of transformation has been performed on Quadrilateral ABCD?
 - d. What do you notice about the measures of the segments and the angles?
4. Look at your three transformations and complete this statement:
In a translation, rotation, or reflection the pre-image and image are _____.



Describing Translations

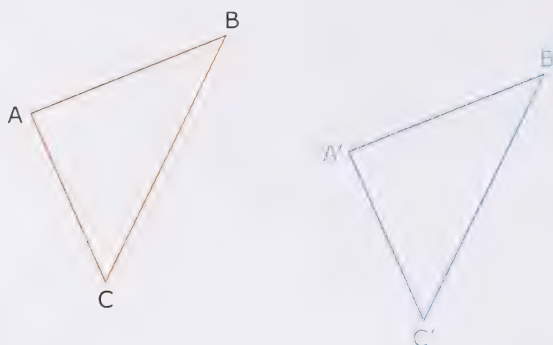
You can describe translations using words and values on the coordinate plane. A translation that moves an image **right 5 units** and **down 7 units** is shown here:



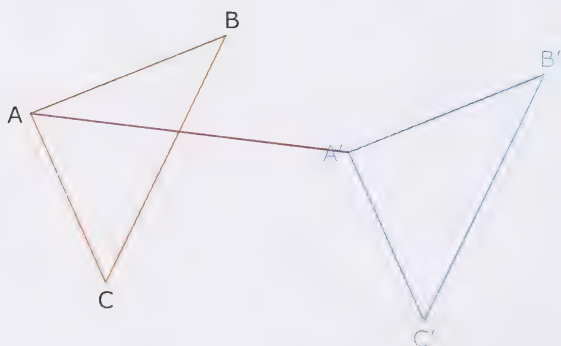
There are ways to describe translations without referring to the coordinate plane. You can draw a line from a vertex on the pre-image to the corresponding vertex on the image. Use a ruler or other measuring device to measure this line. Draw a ray with the same measure and direction. This will describe the movement of the translation. This ray is called the **line of translation**.

Example 1

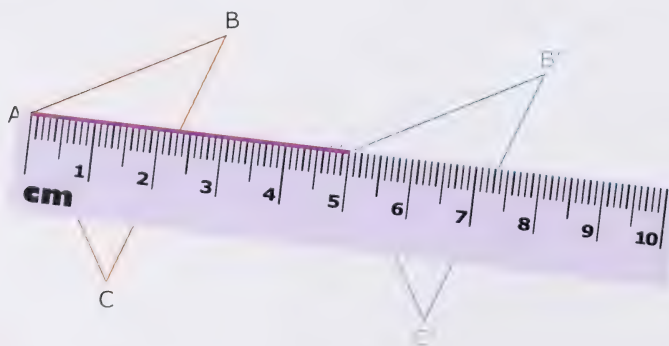
Draw the line of translation of ABC to $A'B'C'$.



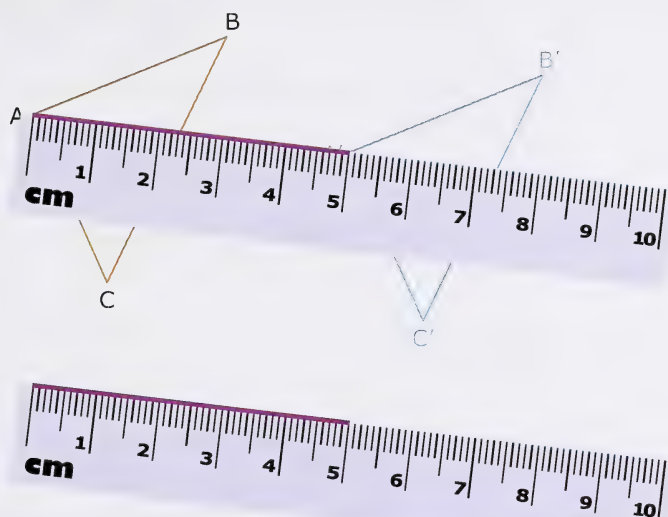
Draw a line segment from A to A' using a straight edge.



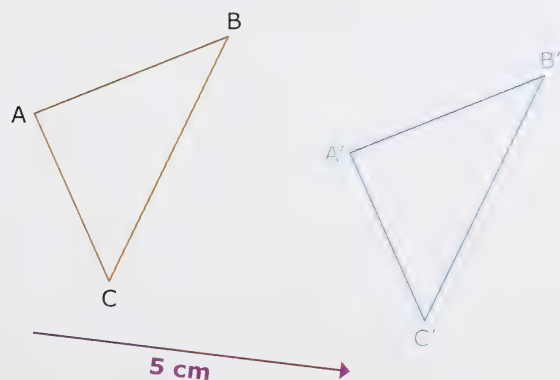
Measure the line segment in centimetres.



Now draw a ray with the same measure and in the same direction beside the pre-image and image.



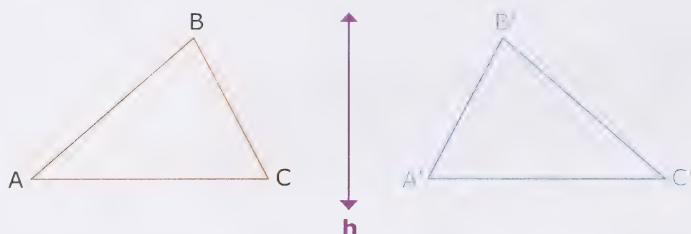
The ray of translation shows you the direction of the translation and the length of the translation. It may be drawn anywhere beside the images.



Describing Reflections

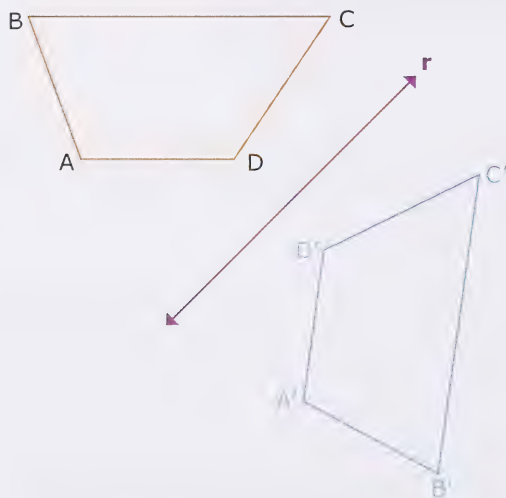
A reflection is described by the line of reflection over which it is reflected. You can name a line using a lower case variable.

A reflection of triangle ABC in the line h :



Example 2

Describe the transformation shown.



Describe the reflection using the name of the pre-image and the line:

Reflection of $ABCD$ in the line r

You can use a MIRA to find lines of reflection between an image and its pre-image. Use the following Exploration to explore reflections.



Let's Explore

Exploration 2: Reflections with a MIRA

Materials: Unit 3, Lesson 8, Exploration 2 page in your Workbook, MIRA, Pencil, Ruler

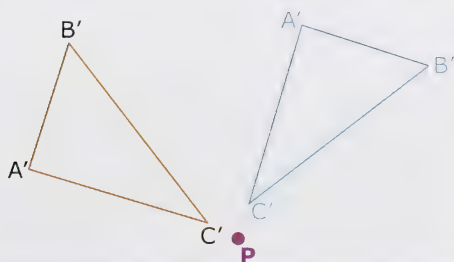
1. Draw a reflection of the figure using a MIRA.
 2. Find the line of reflection of the image and pre-image using a MIRA. Draw the line of reflection and label it **t**.
 3. Use the image and pre-image from number 2. Draw a line between each vertex on the pre-image and its corresponding vertex on the image. Measure the distance from each vertex to the line of reflection.
 4. What do you observe about the distance from the line of reflection to the vertex on the image and pre-image for each vertex?
- Go online to watch the Notepad Tutor Lesson: Describing Rotations around a Fixed Point.

Describing Rotations

A rotation is described by:

- the point of rotation
- the angle of rotation
- the direction of rotation

The following is a 90° clockwise rotation of triangle **ABC** about point **P**:

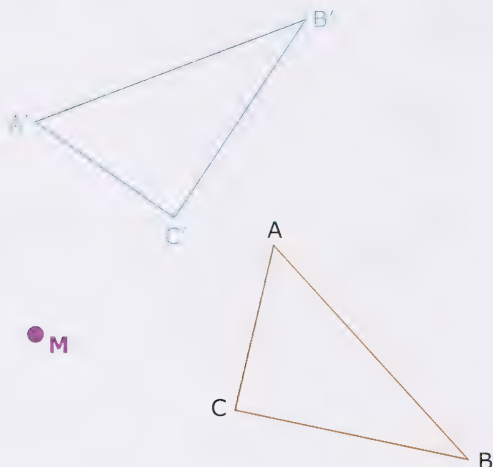


You will need a protractor to measure the angle of rotation. The point of rotation should be labelled on the figure.

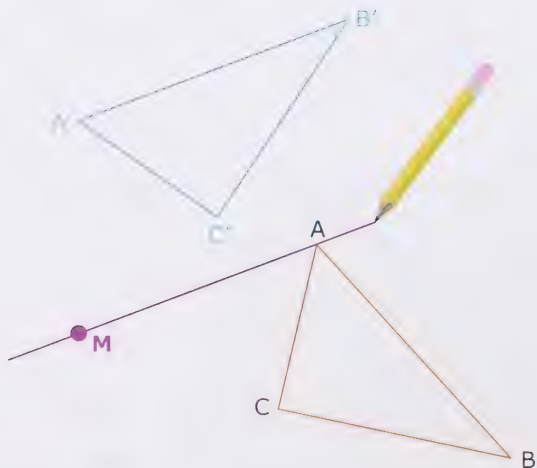
Example 3

Find the angle of rotation for the following.

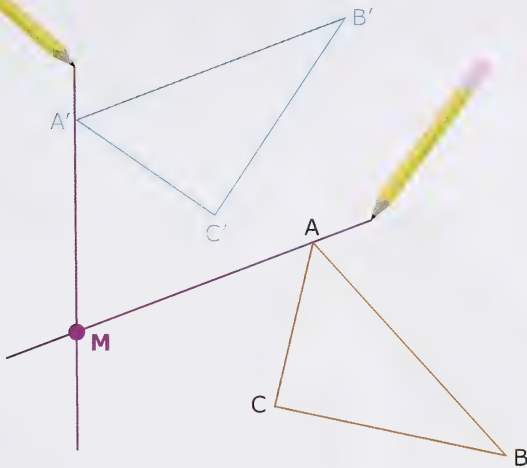
Choose any vertex on the pre-image. Draw a line through the point of rotation and that vertex.



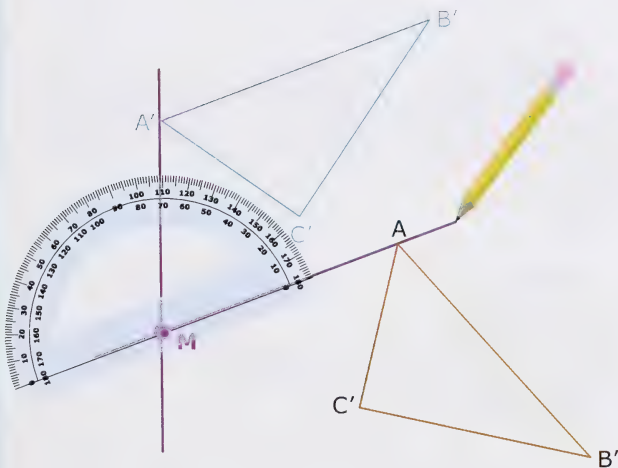
Choose any vertex on the pre-image. Draw a line through the point of rotation and that vertex.



Repeat for the corresponding vertex on the image.



Measure the angle between the two lines you have drawn.



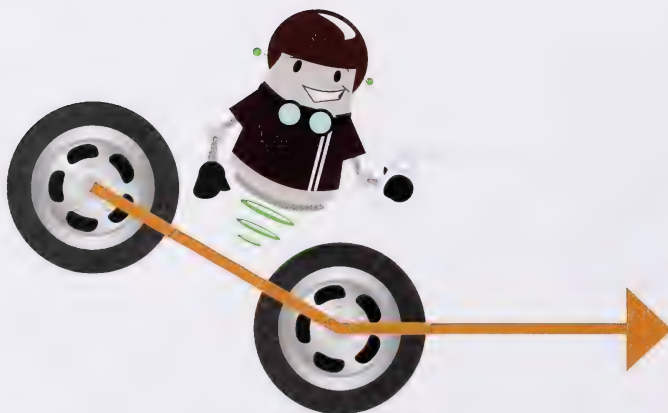
Describe the rotation:

70° counter-clockwise rotation about point **M**



Let's Practice

- Turn in your Workbook to Unit 3, Lesson 8 and complete 1 to 12.

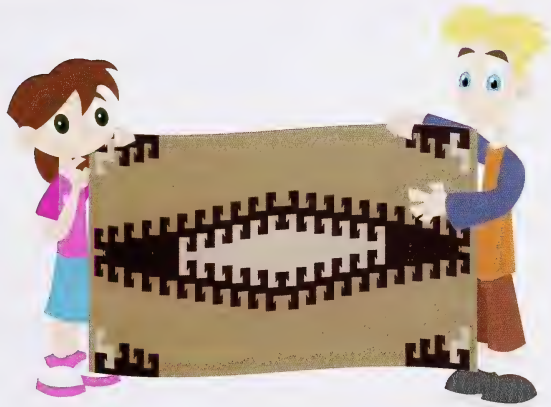


Lesson 9

Combining Transformations

First Nations Art

Some First Nations people use transformations in their artwork. They create beautiful items such as rugs. The rug shown here has one simple geometric shape that has been rotated, translated and reflected to form a pattern.



Reflection

Can you see the shape that is the pre-image?
How many transformations can you identify?



Objectives for this Lesson

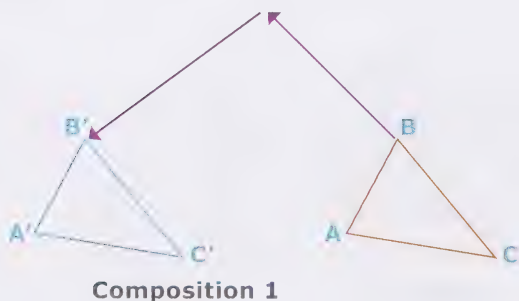
In this lesson you will explore the following concepts:

- Model a given set of successive transformations of a 2-D shape
- Model a given combination of two different types of transformations of a 2-D shape
- Perform and record one or more transformations of a 2-D shape that will result in a given image
- Draw and describe a 2-D shape and its image given a composition

Repeating Transformations

Some transformations may be repeated two or more times to create the final image. This is called a **composition of transformations**.

This image shows a composition of two translations. You can see the line of translation for each on the image:





Let's Explore

Exploration 1: Compositions

Materials: Unit 3, Lesson 9, Exploration 1 page in your Workbook, Internet, Pencil

Use the following activity to create a composition of:

- two reflections
- two translations
- two rotations

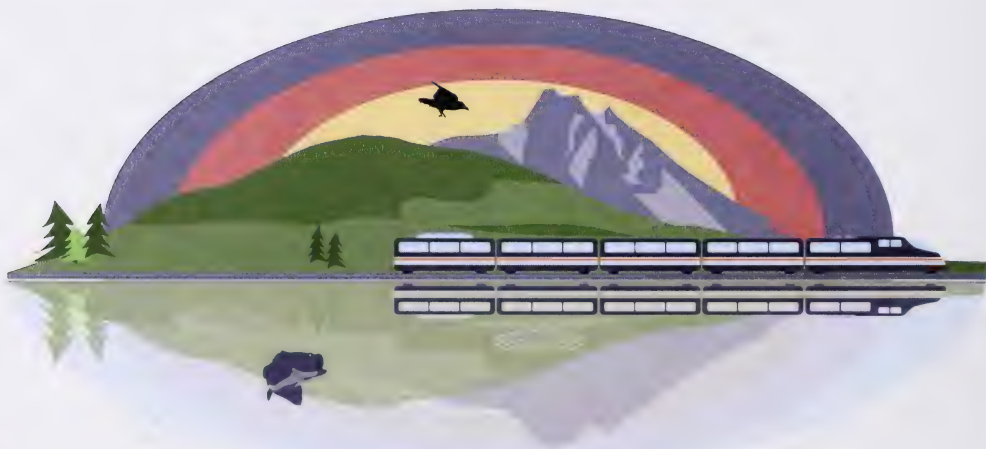
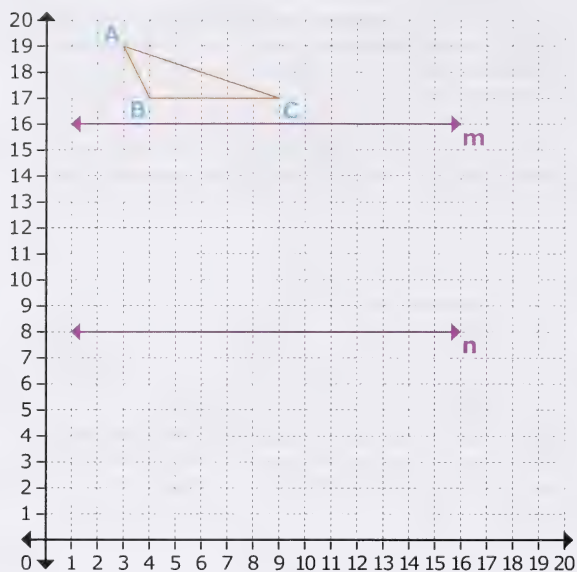
<http://standards.nctm.org/document/eexamples/chap6/6.4/part4.htm#applet>

Answer the following questions after experiencing the activity:

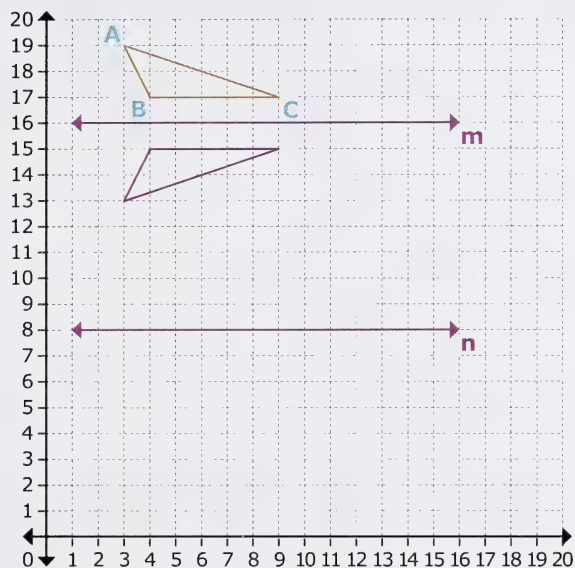
- Does the composition of two reflections in two parallel lines change the orientation of the pre-image?
 - How does the composition of two reflections in intersecting lines resemble another type of transformation?
 - Is it possible for the composition of two translations to change the orientation of the pre-image? Why or why not?
 - What happens if two 90° rotations are performed with the same point of rotation?
- Go online to complete the Concept Capsule: Describing a Combination of Transformations.

Example 1

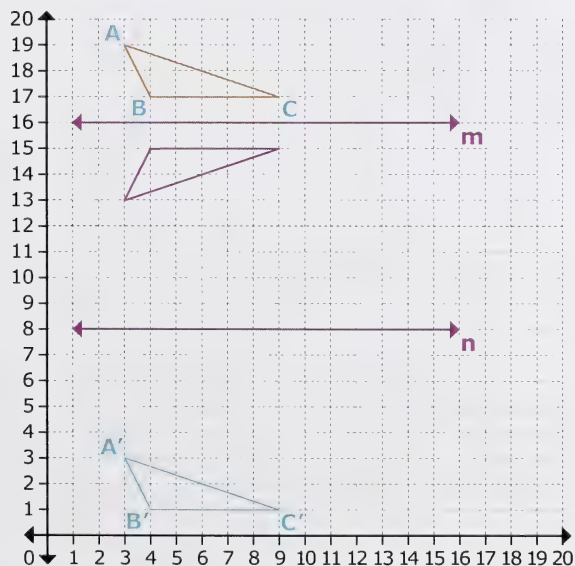
Perform a composition of two reflections in line m and n .



Recall that the reflection will move each point of the pre-image the same distance on the other side of the line of reflection. Reflect in line m first:



Now reflect the image in line n .

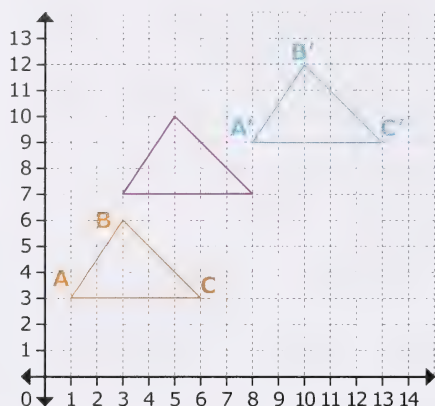


You can do the same with translations.

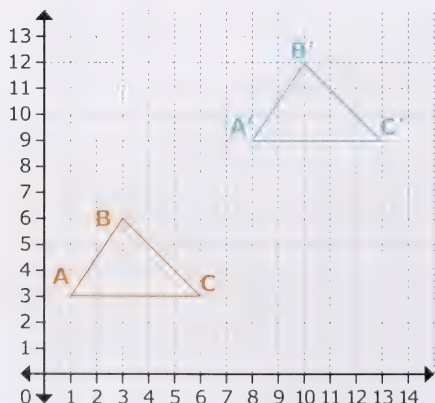
Example 2

Triangle ABC has vertices A (1, 3), B (3, 6), C (6, 3). Find the image of ABC under a translation of right 2 units, up 4 units, followed by a translation of right 5 units, up 2 units.

There are a couple of ways you can do this. You can perform two translations:



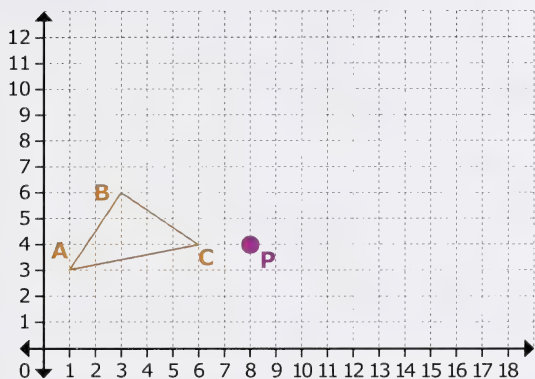
You can also add the two translations together and then perform one translation:
 right 2 up 4 + right 5 up 2 = right 2 + right 5 and up 4 + up 2
 = right 7 and up 6



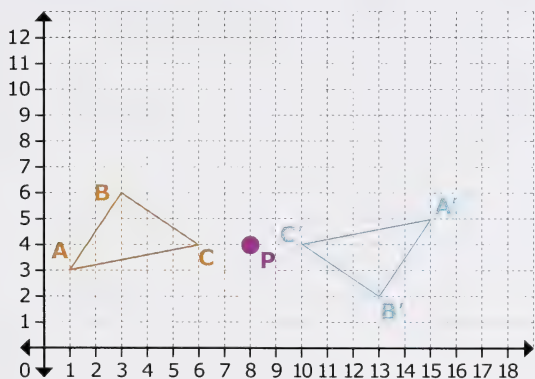
Rotations can also be added as long as the point of rotation is the same.

Example 3

Rotate triangle ABC 90° clockwise about point P followed by a 90° clockwise rotation about point P.



Add the two angles. $90^\circ + 90^\circ = 180^\circ$ so the angle of rotation is 180° . Perform a 180° clockwise rotation.

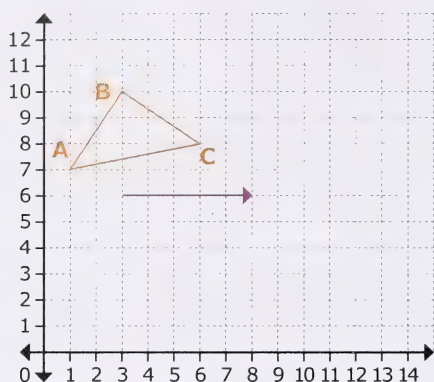


Glide Reflections

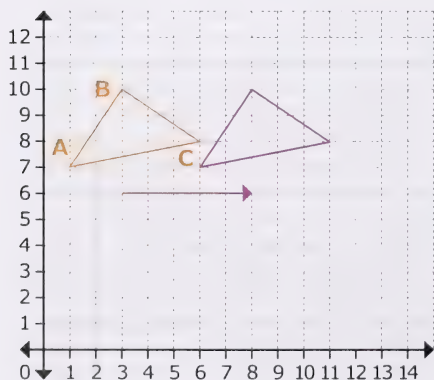
A glide reflection is the composition of a translation followed by a reflection. The ray of translation is also the line of reflection. This ray is called the **slide arrow**.

Example 4

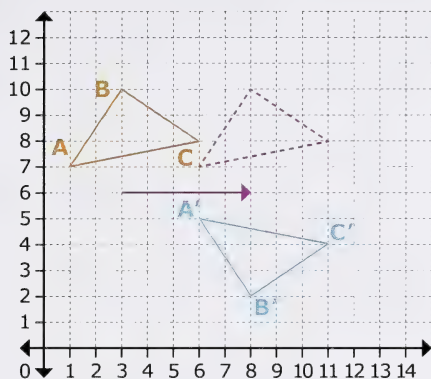
Perform a glide reflection using the given slide arrow.



Either the reflection or translation may be performed first. However, it is often easiest to perform the translation first. The slide arrow is shown as 5 units to the right. Here is the translation:



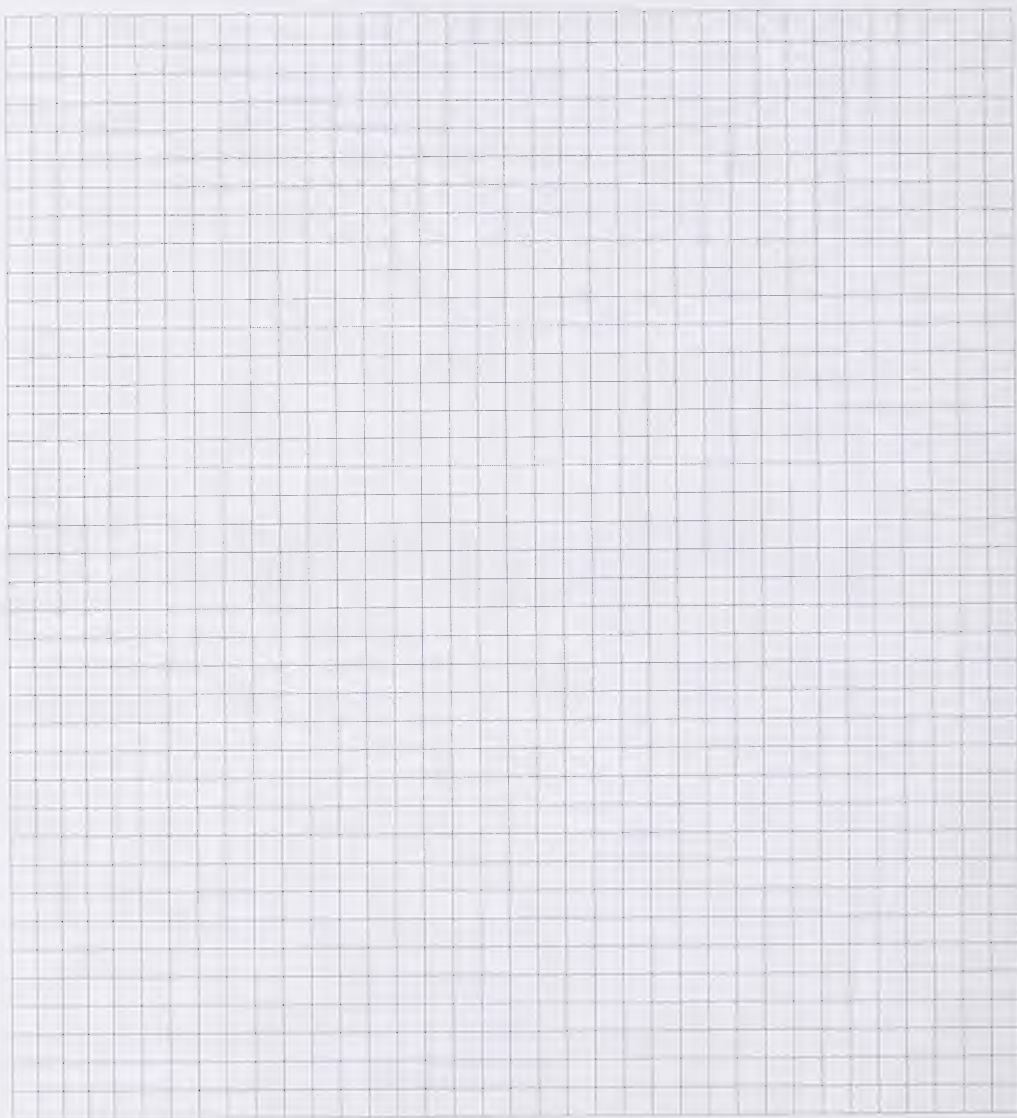
Last, reflect the translated image over the slide arrow:



Let's Practice

- Turn in your Workbook to Unit 3, Lesson 9 and complete 1 to 6.



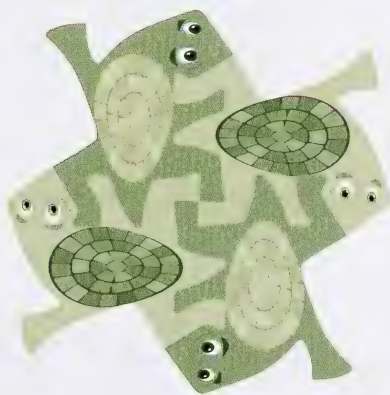


Lesson 10

Transformational Designs

Creating Designs

M.C. Escher was an artist that made remarkable pieces of art using geometric transformations. He was first inspired by the patterns in mosaic tiles. He would create a pre-image and use it to produce a pattern. Here is a sample of a piece of artwork using a pre-image to produce a pattern.



Reflection

Can you see the rotation of the pre-image in this artwork? Can you see the reflection in this artwork?



Objectives for this Lesson

In this lesson you will explore the following concepts:

- Analyze a given design created by transforming one or more 2-D shapes, and identify the original shape(s) and the transformations used to create the design
- Draw and describe a 2-D shape and its image, given a combination of transformations
- Create a design using one or more 2-D shapes, and describe the transformations used

Analyzing Designs

Using block letters you can see how a design can be formed.

Reflection

F F F F F F F F F F F F

Rotation

F F F F F F F F F F F F

Translation

F F F F F F F F F F F F

You can create your own pattern using the letters of your name. Use the following exploration to create your own design from letters.



Let's Explore

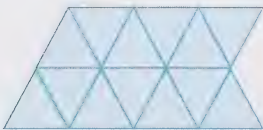
Exploration 1: Create a Letter Pattern

Materials: Unit 3, Lesson 10, Exploration 1 page in your Workbook, Construction Paper, Scissors, Straight Edge, Pencil

1. Create the initials of your name using construction paper.
2. Cut out at least 10 of each letter.
3. Rotate, translate, and reflect your letters to form a pattern.
4. Make a sketch of the pattern you created.

Tessellations

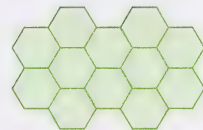
A tessellation is a figure that can cover a surface with no gaps and no overlaps. There are certain polygons that tessellate in a plane.



Equilateral Triangles



Parallelograms

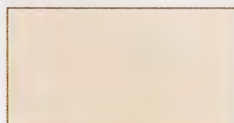


Regular Hexagons

Original designs can be made from these basic tessellating shapes. Other shapes that tessellate:



Square



Rectangle



Rhombus

When Escher created art with tessellations he often began with one shape that would fit together like a puzzle. You can create one of these images. The following example will show you how that is done. You can then create your own.

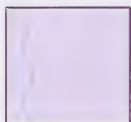
Example 1

Create a unique design that will tessellate.

Start by cutting out a shape that will tessellate. In this case, a square is used:



On one side of the square, draw a pattern that will be cut out:



You can have straight lines or curves on your drawing.

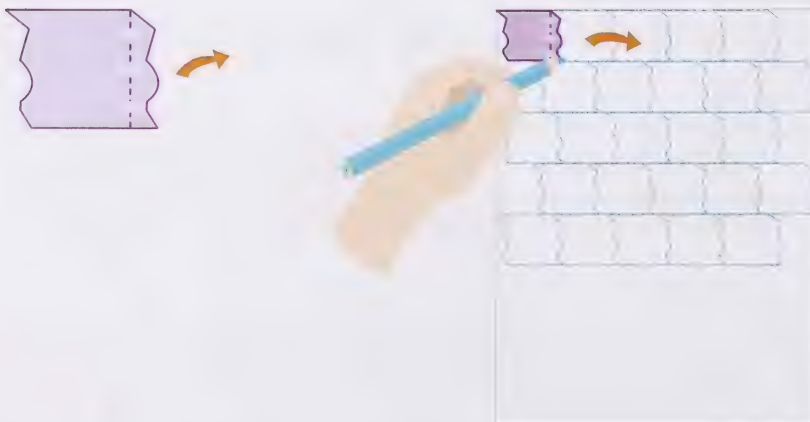
Now cut this out and move it to the opposite side:



Tape the shape on the opposite side.

Now trace your shape on a piece of paper.

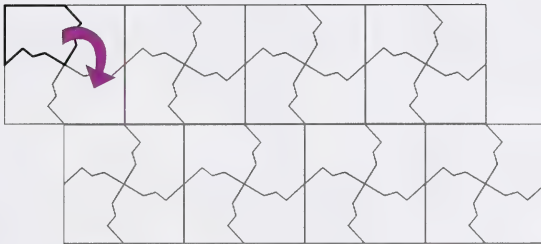
Notice that you can repeat the shape across the paper:



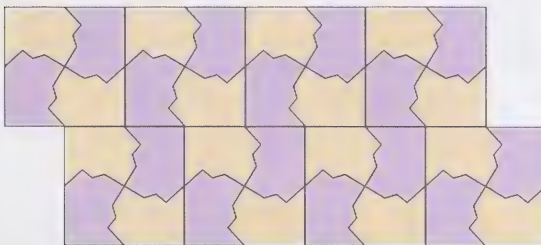
You can also create your own tessellating shapes by rotating the cut out to another side:



When you create a figure through a rotation, the design will rotate within the tessellation:



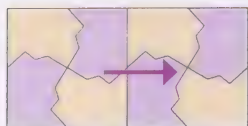
You can add visual interest to a tessellation using colour and art:



In this tessellation you can see a 90° rotation:



You can also see a translation:



Use the following exploration to create your own tessellation.



Let's Explore

Exploration 2: Create a Tessellation

Materials: Unit 3, Lesson 10, Exploration 2 page in your Workbook, Tracing Paper, Pencil Crayons, Scissors, Pencil, Construction Paper, Page of Shapes from the back of this unit in your Workbook

1. Create your own tessellation using one of the shapes from the Exploration page in your Workbook. Use colour to make your shape more interesting.
2. Identify any rotations, translations or reflections in your tessellation. Be sure to hand in your tessellation with your Workbook.



Let's Practice

- Go online to complete the Concept Capsule: Analyzing Tessellations.
- Turn in your Workbook to Unit 3, Lesson 10 and complete 1 to 10.

Lesson 11

Perimeter of Polygons

Regular Polygons in Space

The Mars Exploration Rover A (MER A) was launched on June 5, 2003, followed by the launch of the MER B on June 25, 2003. They both landed on Mars in January of 2004. They were designed to roam the surface of Mars for 90 days but they were still operating as of January 2009. Their job is to gather data and photos to send back to Earth. This allows scientists to study the geology of the planet.

A major question that scientists would like to answer is, "Was there ever water on the surface of Mars?" They can find this by observing the rocks, minerals and geological formations that are on the surface. The MER B discovered some interesting symmetrical structures on the way to the Victoria crater. The stones were shaped like regular polygons.



Reflection

What are some characteristics of a regular polygon?

Objectives for this Lesson

In this lesson you will explore the following concepts:

- Explain how the perimeter of any polygon can be determined
 - Generalize a rule for finding the perimeter of rectangles and squares
 - Write and explain the formula for finding the perimeter of any given rectangle (using patterns and variables)
 - Solve a given problem involving the perimeter of polygons
-
- Go online to watch the Notepad Tutor Lesson: Problem Solving Using the Perimeter of a Rectangle.

Perimeter of Regular Polygons

The perimeter of a polygon is found by adding the measures of each side. You should recall finding the perimeter of figures with side lengths marked on the figure.

Example 1

Find the perimeter of the playground.

To find the perimeter, add the length of all of the sides together.

$$8.1 + 9.2 + 4.5 + 8.6 + 12 = 42.4$$

The perimeter of the playground is **42.4 metres.**



You should be able to make a rule for finding the perimeter of a regular polygon.



Let's Explore

Exploration 1: Perimeter of Regular Polygons

Materials: Unit 3, Lesson 11, Exploration 1 page in your Workbook, Pencil

Find the perimeter of the regular polygons with a side length of 3 cm.

Regular Polygon	Number of Sides	Side Length (cm)	Perimeter (cm)
Equilateral Triangle	3	3	9
Square	4	3	12
Pentagon	5	3	
Hexagon			
Heptagon			
Octagon			
Nonagon			
Decagon			

1. What pattern do you notice in the perimeter of the regular polygons?
2. How would the pattern change if the side length is 4 cm?
3. A regular polygon has 15 sides. The side length is 4 units. What is the perimeter?
4. Write a rule to find the perimeter, **P**, of a regular polygon with **n** sides and a side length of **s**.

Example 2

Find the perimeter of a regular hexagon with side length of 12 metres.

Record the number of sides, n , and the side length, s : $n = 6, s = 12$

Use the rule:

$$P = n \cdot s$$

$$P = (6) \cdot (12)$$

$$P = 72 \text{ metres}$$

The perimeter formula for a regular polygon should help you to find missing side lengths as well as the perimeter.

Example 3

A regular pentagon has a perimeter of 30 centimetres.

What is the length of each side?

Write the parts of the rule that you know:

$$P = 30, n = 5, s = ?$$

Use the rule:

$$P = n \cdot s$$

$$(30) = (5) \cdot s$$

Divide both sides by 5:

$$\frac{30}{5} = \frac{5 \cdot s}{5}$$

Write the units on your answer:

$$6 \text{ cm} = s$$

Perimeter of Rectangles

A special type of quadrilateral is a rectangle.

The Properties of a Rectangle are:

- Opposite sides are congruent
- All angles are right angles



You should be able to find a rule for the perimeter of a rectangle when given the length and width.



Let's Explore

Exploration 2: Perimeter of a Rectangle

Materials: Unit 3, Lesson 11, Exploration 2 page in your Workbook, String, Centimetre Ruler, Scissors, Pencil

1. Cut a length of string that is 42 centimetres long.
2. Create at least six rectangles that have whole number side lengths. Record the measures in the table in your Workbook.
3. Do you notice a relationship between the side lengths and the perimeter?
4. Write a rule for the perimeter, **P**, of a rectangle given the length, **l**, and the width, **w**.

Example 4

Find the perimeter of a rectangle with length of 4.7 cm and width of 2.5 cm.

Write the parts of the rectangle you know:

$$l = 4.7, w = 2.5$$

Use the rule:

$$P = 2l + 2w$$

Use parentheses to show multiplication:

$$P = 2(4.7) + 2(2.5)$$

Simplify:

$$P = 9.4 + 5$$

Write the unit on your answer:

$$P = 14.4 \text{ cm}$$

Solving Perimeter Problems

You should be ready to use your formulas for finding perimeter of regular polygons and rectangles. Solve the problems using your formulas.

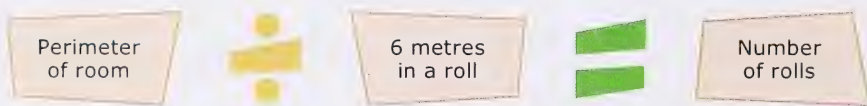
Example 5

Alyssa is putting a border in her room. The walls in her room form a rectangle. The length is 10 metres and the width is 12 metres. The border comes in packages of 6 metres. How many packages of border will she need to buy?

Find the perimeter of the room:

$$P = 2(10) + 2(12) = 20 + 24 = 44 \text{ m}$$

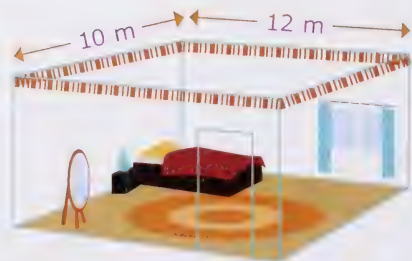
Make a model:



Solve: $44 \div 6 \approx 7.3$

Analyze the answer:

The answer indicates that Alyssa will need **more than 7** rolls. Since the border is sold by the roll, she will need **8 rolls** of border.



Example 6

Daksha and Lian are playing a game that has a board shaped like a regular hexagon. The perimeter of the board is 54 centimetres. What is the length of each side of the game board?

Write the parts of the perimeter rule that you know:
Use the rule:

$$P = 54, n = 6, s = ?$$

$$P = n \cdot s$$

$$(54) = (6) \cdot s$$

Divide both sides by 6:

$$\frac{54}{6} = \frac{6 \cdot s}{6}$$

Write the unit on the answer:

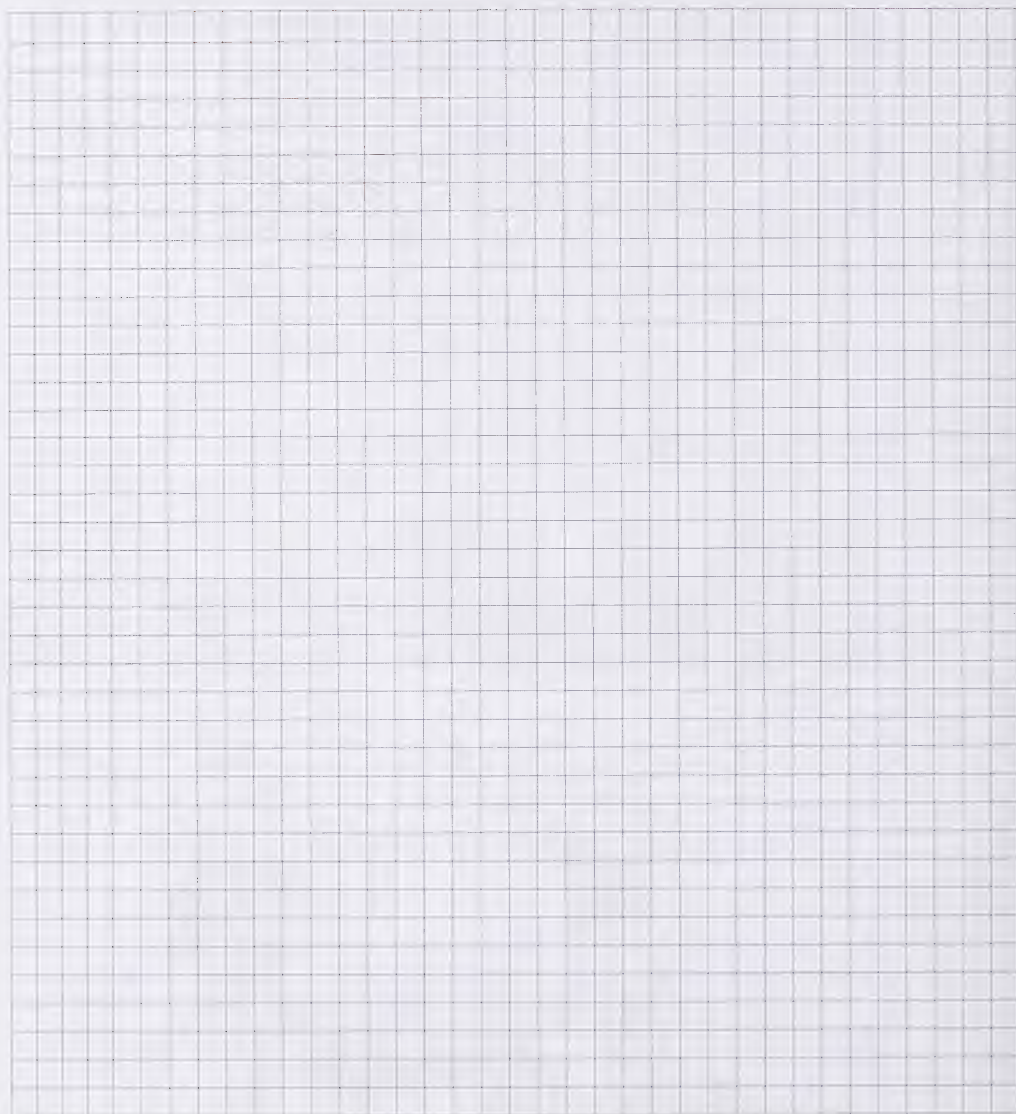
$$9 \text{ cm} = s$$

Each side of the hexagon game board is 9 cm long.



- Turn in your Workbook to Unit 3, Lesson 11 and complete 1 to 28.





Lesson 12

Area of Rectangles

Lawn Care

Many people like to have a nice green lawn. It takes work to make this happen. You need to start with grass that is sprouted from seeds or by laying sod. You need to water and fertilize your grass to make it grow and gain the beautiful green colour.

Fertilizers are often spread about the lawn using a spreader. They are administered based on the area of your lawn. To know how much fertilizer to use, you need to estimate the area of your lawn. Many lawn fertilizers use a ratio of 1 kg of fertilizer for every 100 square metres of lawn.



Reflection

Could you estimate the area of your lawn? Could you estimate the area of the lawn at your school?

Objectives for this Lesson

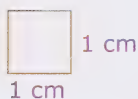
In this lesson you will explore the following concepts:

- Generalize a rule for determining the area of rectangles
- Write and explain the formula for finding the area of any given rectangle
- Explain how the area of any rectangle can be determined
- Solve a problem involving the area of rectangles

Area of Rectangles

The area of a shape is the amount of space that it covers.
Area is measured in square units.

Figure



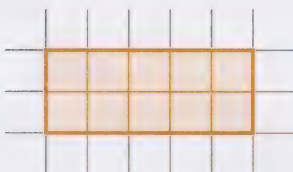
Notation for Area

1 cm^2

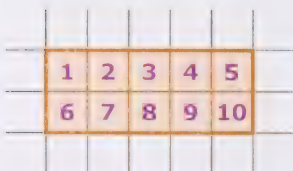
OR

1 square centimetre

Use the grid paper to find the area of the given rectangle



The squares of the grid paper are each one square unit.
You can count the number of squares within the figure to find the area.



The area is 10 square units.



Let's Explore

Exploration 1: Generalize Area of a Rectangle

Materials: Unit 3, Lesson 12, Exploration 1 page in your Workbook, Square Tiles, or Grid Paper from the back of this unit in your Workbook, Pencil

1. Make rectangles on your Grid Paper, or with Square Tiles, using the side lengths shown in the table. Record the area of each rectangle.

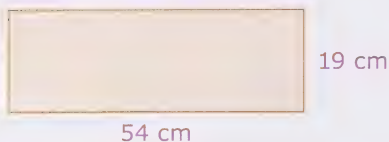
Length	Width	Area (square units)
2	4	
3	4	
4	4	
5	4	
6	4	
3	5	
3	6	
3	7	
3	8	
3	9	
3	10	

2. Do you notice a relationship between the area of the rectangle and the side lengths?
3. Write a rule for the area of a rectangle using the length, l , and the width, w .
4. Test your rule using a rectangle with a length of 8 units and a width of 4 units.

You can use your rule to find the area of any given rectangle. You can also use the rule to find a missing side length of a rectangle, when you know its area.

Example 1

Find the area of the given rectangle.



Record the length and width of the rectangle: $l = 54 \text{ cm}$, $w = 19 \text{ cm}$

Use the rule you found for area of a rectangle: $A = l \cdot w$

$$A = (54) \cdot (19)$$

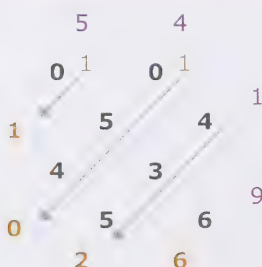
You may need to use a strategy for multiplying large numbers.
Here is a review of the diagonal method:

Multiply

$$\begin{array}{r} 54 \\ 00 \\ \hline 54 \\ 430 \\ \hline 1026 \end{array}$$

Add Diagonals

(Remember to carry tens)



The diagram shows the multiplication 54 x 19 using the diagonal method. The digits are arranged in a grid. Diagonal arrows point from the top-left to the bottom-right, adding the products of digits that lie on the same diagonal. The diagonals are labeled with their position (0, 1, 2, 3, 4). The final result, 1026, is written along the bottom-left diagonal.

The area is 1 026 square centimetres.

Example 2

Find the area of a rectangle with length of 6 cm and width of 7 cm.

Write the known sides: $l = 6, w = 7$

Use the formula: $A = l \cdot w$

$$A = (6) \cdot (7)$$

$$A = 42 \text{ cm}^2$$

Example 3

Find the length of a rectangle with an area of 48 cm^2 and a width of 12 cm.

Write the parts of the rule that you know:

$$A = 48, l = ?, w = 12$$

Replace the variables with the values they represent:

$$A = l \cdot w$$

$$(48) = l \cdot (12)$$

Divide both sides by 12:

$$\frac{48}{12} = \frac{l \cdot 12}{12}$$

$$4 = l$$

Write the units:

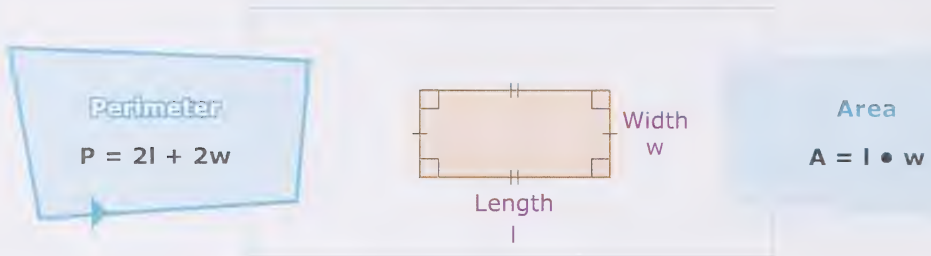
$$4 \text{ cm} = \text{length}$$



Perimeter and Area of Rectangles

The rules you used to find the perimeter and area of rectangles can also be called a formula. You can use formulas to simplify the process of measuring a figure.

Formulas For a Rectangle



Use the perimeter and area formulas to answer the following questions.

Example 4

Daksha's room has a perimeter of 36 units and an area of 80 square units. What are the lengths of the walls in Daksha's room?

The area of the room is found using the formula: $A = l \cdot w$

Since you know the area is 80 square units you can list the possible wall lengths based on the area:

1, 80 2, 40 4, 20 5, 16 8, 10

Now you can use trial and error to find the pair that fits the perimeter:

$$P = 2l + 2w$$

Lesson 12: Area of Rectangles

$$P = 2l + 2w$$

$$1, 80: P = 2(1) + 2(80) = 162 \quad \times$$

$$2, 40: P = 2(2) + 2(40) = 84 \quad \times$$

$$4, 20: P = 2(4) + 2(20) = 48 \quad \times$$

$$5, 16: P = 2(5) + 2(16) = 42 \quad \times$$

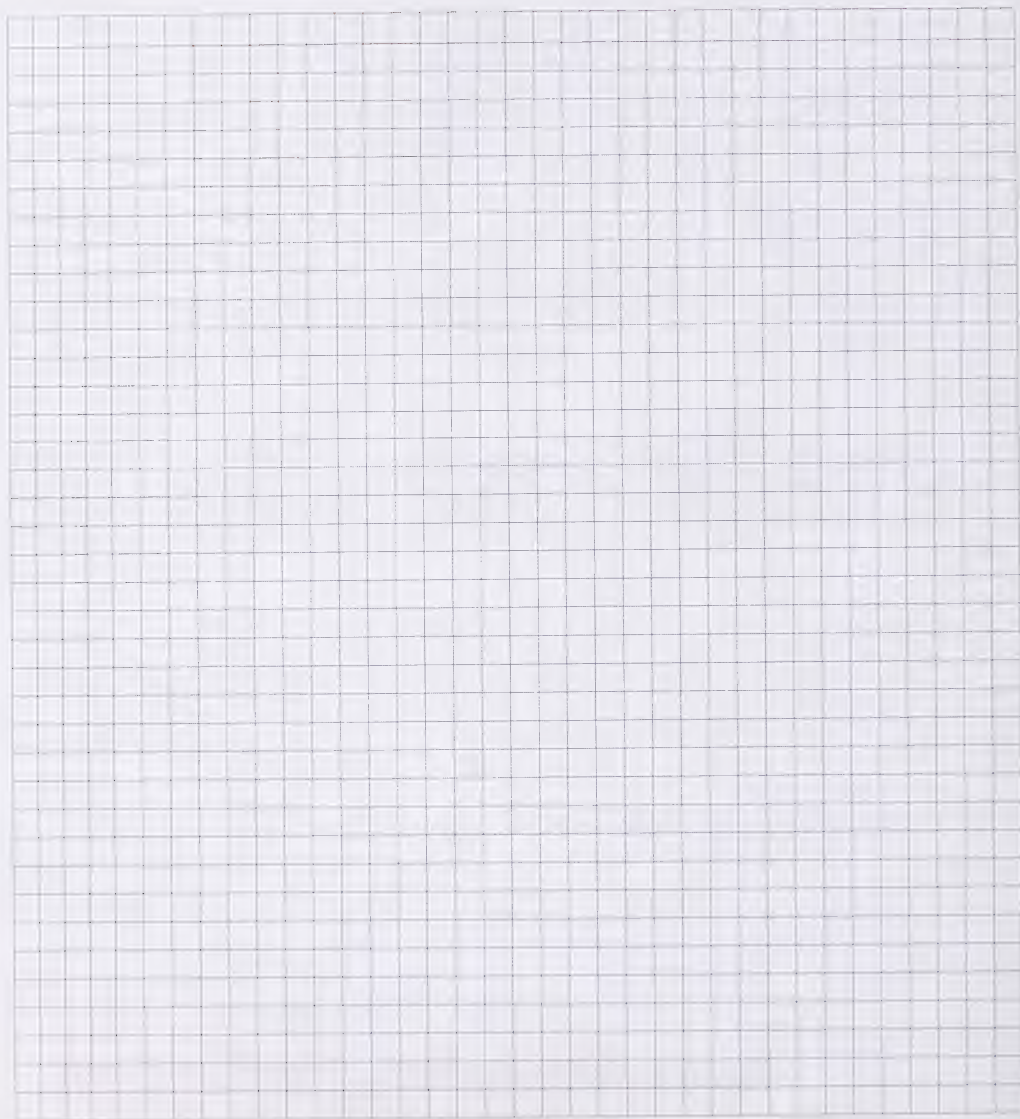
$$8, 10: P = 2(8) + 2(10) = 36 \quad \checkmark$$

The lengths of the walls are 8 units and 10 units.



- Turn in your Workbook to Unit 3, Lesson 12 and complete 1 to 16.
- Go online to complete the Concept Capsule: Area of Rectangles vs. Area of Parallelograms.





Lesson 13

Volume

Aquariums

When you think of an aquarium, you probably think of fish in a tank in someone's home. This is a common type of aquarium. There are also really large aquariums that hold large marine life. The Vancouver Aquarium is the home of a baby beluga whale named Tiqa.

Tiqa and her mother Qila



The Vancouver Aquarium boasts the Pacific Canada Pavilion. The indoor tank holds over 260 000 litres of water.



Reflection

How many four litre milk jugs full of water would it take to fill Tiqa's tank?

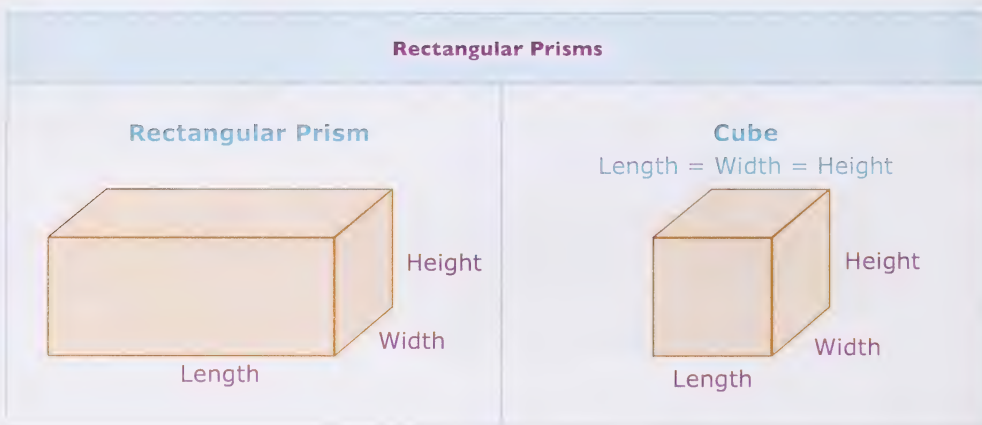
Objectives for this Lesson

In this lesson you will explore the following concepts:

- Generalize a rule for determining the volume of rectangular prisms
- Explain how the volume of any rectangular prism can be determined
- Solve a given problem involving the volume of right rectangular prisms

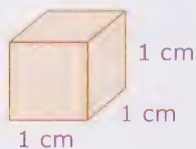
Rectangular Prisms

A **rectangular prism** is a solid that has two rectangular bases and all its other sides are rectangles as well. A rectangular prism has three **dimensions**: length, width and height. A special rectangular prism is a cube. All three dimensions of a cube are the same.

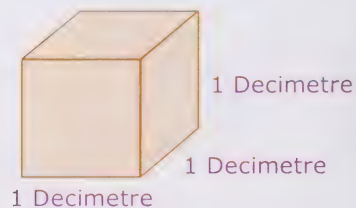


The **volume** of a solid figure is the amount of space that it occupies. Volume is measured using unit cubes:

1 Centimetre Cube

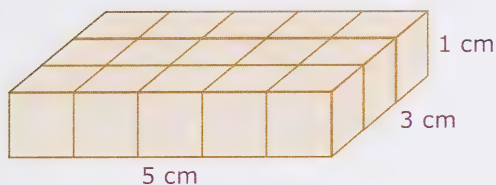


1 Decimetre Cube



You can describe the volume using the following notations:

Rectangular Prism



Notation for Volume

15 cubic centimetres

OR

15 cm³



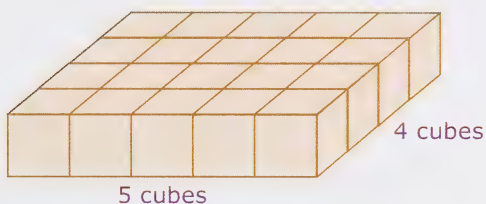
Let's Explore

Exploration 1: A Rule for Volume

Materials: Unit 3, Lesson 13, Exploration 1 page in your Workbook, Centimetre Cubes, or Grid Paper from the back of this unit in your Workbook, Pencil

1. Build a rectangular prism that looks like this:

20 Total Cubes



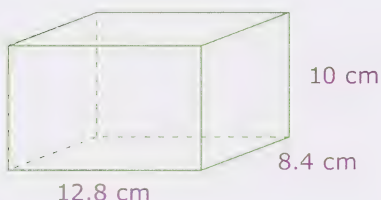
2. Record the volume in the chart. Make all rectangular prisms that you need to make to complete the volume of each in the chart.
3. Do you notice a pattern to the volume?
4. How does the pattern relate to the length and width?

5. How does the volume relate to the dimensions?
6. Make several rectangular prisms of your own. Record the dimensions and the volume of each rectangular prism in the chart.
7. How does the volume relate to the dimensions?
8. Write a rule to find the volume of a rectangular prism given the dimensions. Use V for volume, l for length, w for width, and h for height.
9. Test your rule by finding the volume of a rectangular prism with the following dimensions: $l = 2$, $w = 4$, $h = 3$

The rule you wrote for the volume of a rectangular prism can also be called a **formula**. Use the formula to find the volume of rectangular prisms or to find missing dimensions.

Example 1

Find the volume of the rectangular prism.



Record the length, width and height:

$$l = 12.8, w = 8.4, h = 10$$

Complete the formula using the dimensions:

$$V = l \cdot w \cdot h$$

$$V = (12.8) \cdot (8.4) \cdot (10)$$

Hint: Multiply 10 times a decimal first to make multiplication easier.

$$V = 12.8 \cdot 84$$

$$V = 1\,075.2$$

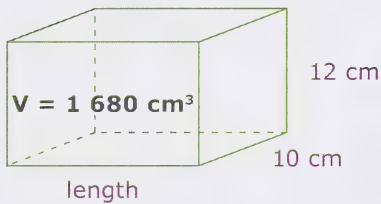
Write the units on your answer:

$$V = 1\,075.2 \text{ cm}^3$$

- Go online to watch the Notepad Tutor Lesson: Determining Volume Using a Formula.

Example 2

Use the formula for a rectangular prism to find the missing dimension of the given figure.



Record the dimensions:

$$l = ?, w = 10, h = 12, V = 1\,680$$

Use the formula.

$$V = l \cdot w \cdot h$$

Replace all variables with the values they represent:

$$(1\,680) = l \cdot (10) \cdot (12)$$

Simplify the expression on the left side:

$$1\,680 = l \cdot 120$$

Solve the equation by dividing by 120:

$$\frac{1\,680}{120} = \frac{l \cdot 120}{120}$$

$$14 = l$$

Write the units in your answer:

$$14\text{ cm} = l$$

Solving Problems

You can use the formula for finding the volume of a rectangular prism to solve problems.

Example 3

Nina just bought a fish tank. The length of the tank is 50 cm. The width is 43 cm and the height is 51 cm. 1 litre is equal to 1 000 cubic centimetres. About how many litres of water will Nina need to fill the tank?



Record the length, width and height:

$$l = 50, w = 43, h = 51$$

Complete the formula using the dimensions:

$$V = l \cdot w \cdot h$$

$$V = (50) \cdot (43) \cdot (51)$$

$$V = 2\,150 \cdot 51$$

$$V = 109\,650$$

Write the units on your answer:

$$V = 109\,650 \text{ cm}^3$$

Estimate the number of litres:

Change the volume to a comparable number:
110 000 cm³

THINK

$$110\,000 \div 1\,000 = 110$$

Nina needs about 110 litres of water to fill the tank.

Example 4

Cameron fills his fish tank to a height of 18 cm. How many more cubic centimetres of water would it take to completely fill the tank?



The water in the tank has a height of 18 cm.

Record the dimensions of the water:

$$l = 60, w = 40, h = 18$$

Find the volume of the water:

$$V = l \cdot w \cdot h$$

$$V = (60) \cdot (40) \cdot (18)$$

$$V = 43\,200 \text{ cm}^3$$

Find the volume of the tank:

$$l = 60, w = 40, h = 50$$

$$V = l \cdot w \cdot h$$

$$V = (60) \cdot (40) \cdot (50)$$

$$V = 120\,000 \text{ cm}^3$$

Find the difference between the volume of the tank and the volume of the water:

$$120\,000 - 43\,200 = 76\,800 \text{ cm}^3$$

Cameron needs $76\,800 \text{ cm}^3$ more water to fill the tank.

Example 5

Cameron's sister has a fish tank also. Her tank has a volume of $37\,500\text{ cm}^3$. Find the area of the lid for the tank.



The lid for the tank is a rectangle that would have the length and the width as its sides.

Find the missing width of the tank:

$$l = 50, w = ?, h = 30, V = 37\,500$$

$$V = l \cdot w \cdot h$$

$$(37\,500) = (50) \cdot w \cdot (30)$$

$$37\,500 = 1\,500 \cdot w$$

$$\begin{array}{r} 37500 \\ \div 1500 \\ \hline 25 \end{array} = \begin{array}{r} 1500w \\ \div 1500 \\ \hline w \end{array}$$

$$25\text{ cm} = w$$

Now find the area of the rectangle lid using $l = 50$ and $w = 25$:

$$A = l \cdot w$$

$$A = (50) \cdot (25)$$

$$A = 1\,250\text{ cm}^2$$

The area of the lid for the tank is $1\,250\text{ cm}^2$.



- Turn in your Workbook to Unit 3, Lesson 13 and complete 1 to 15.

Sources - Student Learning Guide

Lesson 1

Page 3-1: Parliament buildings,
http://www.ottawakiosk.com/parliament/p_2.html, http://www.parl.gc.ca/publications/Parl_Bldgs/ParlBldgs-e.asp, http://www.ottawakiosk.com/parliament_buildings.html

Lesson 4

Page 3-25: Art Sculptures,
<http://www.citywindsor.ca/000356.asp?artist=lazarus1>, <http://www.netpotential.com/lazarus/OutdoorMetal/consophialink.html>

Lesson 5

Page 3-33: Triangles and the Biosphère, http://www.biosphere.ec.gc.ca/The_Sphere-WS73DCCD17-1_En.htm

Lesson 6

Page 3-41: Geometric Art,
<http://www.mcescher.com>

Lesson 7

Page 3-47: Symmetry in Architecture,
<http://www.tajmahal.org.uk>

Lesson 10

Page 3-81: Creating Designs,
<http://www.mcescher.com>

Lesson 11

Page 3-87: Regular Polygons in Space,
<http://www.thunderbolts.info/tpod/2007/arch07/071012marsminiature.htm>, http://marsrover.nasa.gov/mission/tl_launch.html, Second image: <http://photojournal.jpl.nasa.gov/catalog/PIA08813>, Third image: <http://photojournal.jpl.nasa.gov/catalog/PIA10077>

Lesson 12

Page 3-95: Lawn Care,
<http://www.qc.ec.gc.ca/ecotrucs/solutionsvertes/lawncare.htm>

Lesson 13

Page 3-103: Aquariums,
<http://www.vanaqua.org/home/>

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